An Investigation Of The Improved Nyquist Pulses Families For OFDM Use

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Abstract: **The new families of Nyquist pulses recently proposed and studied [15, 16, 17, 18] are investigated for the OFDM use, to reduce the sensitivity of the OFDM systems to the frequency offset. The results presented in this paper are comparable or outperform the recently found pulses [2, 3, 4, 5, 8, 9] in terms of the intercarrier interference (ICI) power.**

I. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) is a multi-carrier transmission technique used in digital communications systems. This technique is built on the principle that the modulated signals are orthogonal so they do not interfere with each other.

This paper is focused on the problem of reducing the ICI power in transmission over OFDM systems. OFDM is very sensitive to carrier frequency offset caused by the jitter of carrier wave and phase errors between the transmitter and receiver.

Recent papers have reported and examined new families of pulses which are intersymbol interference (*ISI*)- free [15], [16], [17], [18]. In the sequel we present and examine the employment of new *ISI*-free pulses in an OFDM system.

II. SYSTEM MODEL AND ICI ANALYSIS

The complex envelope of one radio frequency (RF) *N*-subcarrier OFDM block with pulse-shaping [10] is expressed as:

$$x(t) = e^{j2\pi f_c t} \sum_{k=0}^{N-1} a_k\, p(t) e^{j2\pi f_k t} \tag{1}$$

where: $j = \sqrt{-1}$, f_c is the carrier frequency, f_k is the subcarrier frequency of the *k*-th subcarrier, $p(t)$ is the time-limited pulse shaping function and a_k is the data symbol transmitted on the *k*-th subcarrier and has mean zero and normalized average symbol energy; data symbols are uncorrelated.

Frequency offset, $\Delta f\ (\Delta f \geq 0)$, and phase error θ, are introduced during transmission because of channel distortion or receiver crystal oscillator inaccuracy.

The average ICI power, averaged across different sequences [8] is:

$$\overline{\sigma^{m}_{ICI}} = \sum_{\substack{k \neq m \\ k=0}}^{N-1} \left| P\left(\frac{k-m}{T}\right) + \Delta f \right|^2 \tag{2}$$

The average ICI power depends not only on the desired symbol location m, and the transmitted symbol sequence, but also on the pulse-shaping function at the frequencies $\left(\left((k-m)/T\right) + \Delta f\right),\ k \neq m,\ k = 0,1,\ldots,N-1$ and the number of subcarriers.

The ratio of average signal power to average *ICI* power is denoted *SIR* and expressed in equation (3).

$$SIR = \left|P(\Delta F)\right|^2 / \sum_{\substack{k \neq m \\ k=0}}^{N-1} \left|P((k-m)/T) + \Delta f\right|^2 \tag{3}$$

III. A NEW FAMILY OF PULSE SHAPES

The proposed new family of pulses are generated by low-pass filter with odd symmetry about the corresponding ideally band-limited cutoff frequency.[15], [16],[18].

In [15] a new family of Nyquist pulses has been proposed, which is defined as

$$G_i(f) = \frac{1}{2B^i \alpha^i}(B - f)^i + \frac{1}{2} \tag{4}$$

For i - odd the pulses show odd symmetry around *B* and their definition can be:

$$S_i = \begin{cases} 1, & |f| \leq B(1-\alpha) \\ G(f), & B(1-\alpha) \leq |f| \leq B(1+\alpha) \\ 0, & B(1+\alpha) < |f| \end{cases} \tag{5}$$

For i - even, the *flipped- technique* is used and they are denoted *flipped-* $G(f)$ [4].

In [16] the Nyquist filter characteristic is obtained from combining two types of characteristics with odd-symmetry. Here $G(f)$ is the *flipped-exponential* characteristic proposed in [2] and $H_i(f)$ is the family of parabolic, cubic and quartic ramps proposed in [15].

$$G(f) = e^{f} \tag{6}$$

$$H_i(f) = -(1)^i \frac{(f - B)^i}{B^i b^i}\left(e^{\frac{\ln 2(b-\alpha)}{\alpha}} - \frac{1}{2}\right) + \frac{1}{2} \tag{7}$$

For i - odd the pulses show odd symmetry around B and their definition is:

$$S_i(f) = \begin{cases} 1, & |f| \le B(1-\alpha) \\ G(f), & B(1-\alpha)| \le |f|| \le B(1+b) \\ H_i(f) & B(1-b)| \le |f|| \le B(1+b) \\ G(f) - \mathit{flipped} & B(1+b)| \le |f|| \le B(1+\alpha) \\ 0, & B(1+\alpha) < |f| \end{cases} \tag{8}$$

For i - even, the vestigial symmetry is obtained by choosing $H_i(f)$ for the frequency interval $B(1-\alpha) \le f \le B$ and $1 - H_i(f)$ for $B \le f \le B(1+\alpha)$.

The expressions were derived imposing continuity conditions at $f = B(1-b)$ and $f = B(1+b)$ and a value of 0.5 at $f = B$. This technique is denoted in the sequel as *double-ramp flipped-*$G(f)$.

In [18] it is proposed a linear combination between two pulses $p_1(t)$ and $p_2(t)$ [6].

The linear combination of two pulses guarantees that the resulting pulse has a bandwidth not greater than that of constituent pulse with larger bandwidth, and if the constituent pulses are ISI-free, then the resulting pulse will also be ISI-free [5].

In the first case, the linear combination technique is applied to the pair of pulses built from the *flipped-exponential* (FE) pulse for $p_1(t)$ and $s_{i=2,3,4}(t)$ [15] for $p_2(t)$. The new pulses are obtained as a result of the linear combination.

$$p_i(t) = q p_{FE}(t) + (1-q) s_i(t), \qquad i = 2,3,4 \tag{9}$$

In the second case, the pair of ISI-free pulses is built from $s_{i=2,3,4}(t)$ for $p_1(t)$ and *raised-cosine* (RC) pulse [1], [2] for $p_2(t)$. The new pulses obtained as a result of the linear combination for this case are given by:

$$r_i(t) = q s_i(t) + (1-q) p_{RC}(t), \qquad i = 2,3,4 \tag{10}$$

IV. SIMULATION RESULTS

The new families of Nyquist pulses show reduced maximum distortion, a more open receiver eye and decreased symbol error probability [4], [12] in the presence of timing error, as compared with the *flipped-exponential* (FE) pulse [2] with the same roll-off factor. Its transmission properties were thoroughly investigated and show that the pulses have practical importance.

The proposed and studied pulses can also be used for reducing average *ICI* power in OFDM systems [8].

In the sequel we followed the same model as in [19] to evaluate the average *ICI* power and the average signal power to average *ICI* power ratio (*SIR*). The simulations results are obtained for the 64-subcarrier OFDM systems.

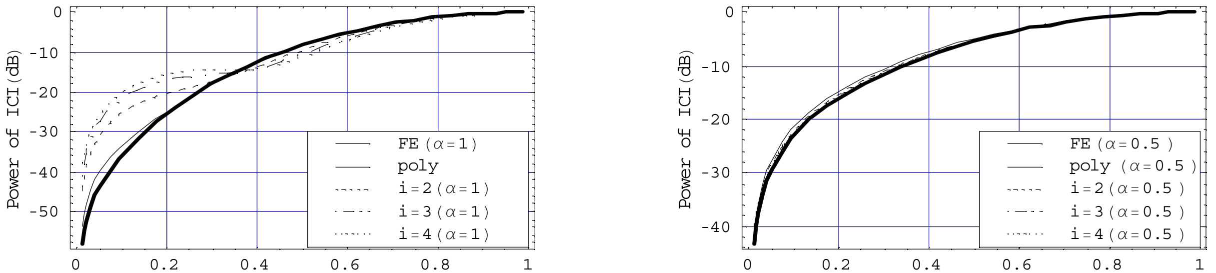
The impulse responses reveal that new pulses defined by $s_i(t)$ [15] with *i=2* follows closely the *flipped-exponential* pulse. Regarding the other pulses, the decrease of the first side lobe is more significant. In Fig. 1. a) and Fig. 1. b) is plotted the average ICI power for the pulses studied in [15] and *flipped-exponential* pulse taken as reference. The ICI power for the new pulses is smaller than ICI power for *flipped-exponential* pulse. The results are for the case when $\alpha = 1$ and $\alpha = 0.5$. The increasing of α is expecting to lead up to the reduction of ICI power. An increasing of α corresponds to reducing the side lobes in the spectrum.

We observe that the studied pulses show better results than *flipped-exponential* pulse and similar results with polynomial pulses [9], as shown in Fig. 1. a), which presents the ICI power for roll-off factor $\alpha = 1$. We observe that the behavior of the pulses is different from that of pulses taken as a reference. When normalized frequency offset is bigger than 0.35 the proposed pulses outperform the references pulses in terms of average ICI power.

If $\Delta f = 0.48$ then average ICI power is -10.2726dB, -11.283dB and -12.0109dB for *i=2, i=3* and *i=4* respectively. The s_4 pulse achieves 2.84587dB and 3.01067dB smaller ICI power than *flipped-exponential* pulse and polynomial pulse, respectively. The s_2 pulse achieves 1.1076dB and 1.2724dB smaller ICI power than *flipped-exponential* pulse and polynomial pulse, respectively. The s_3 pulse achives 2.11804dB and 2.28284dB smaller ICI power than *flipped-exponential* pulse and polynomial pulse, respectively.

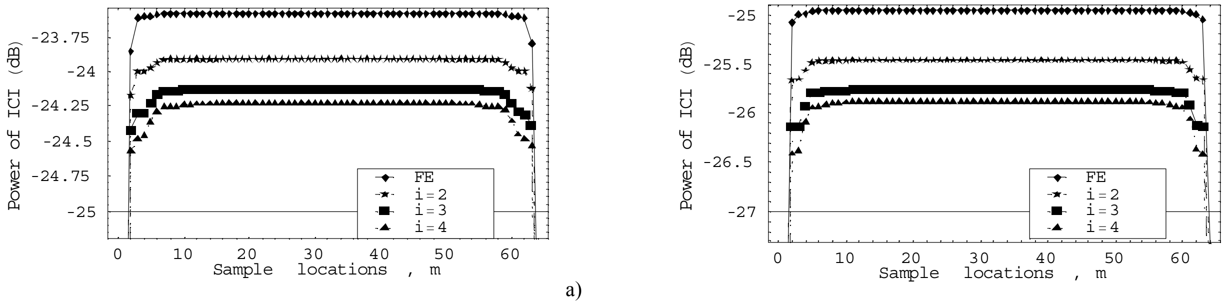
The variations of average ICI power with sample location *m* are presented in Fig. 2. a) and 2. b) for $\alpha = 0.25$ and $\alpha = 0.35$ respectively. The normalized frequency offset is $\Delta f T = 0.05$. Comparing Fig. 2. a) with Fig. 2. b) we observe the reductions of ICI power due to the increase of the roll-off factor α. As expected, the *ICI* drops for the samples located near sample locations 0 and *N-1*, because these samples have fewer interfering samples. The proposed pulses perform better than considered *flipped-exponential* pulse, as seen in the Fig. 1.

When we concern on the comparative performance of this pulses in terms of the average signal power to average *ICI* power ratio denoted as *SIR* in a 64-subcarrier OFDM system [19], the results are plotted in Fig. 3. a), Fig. 3. b) and Fig. 3. c), respectively.



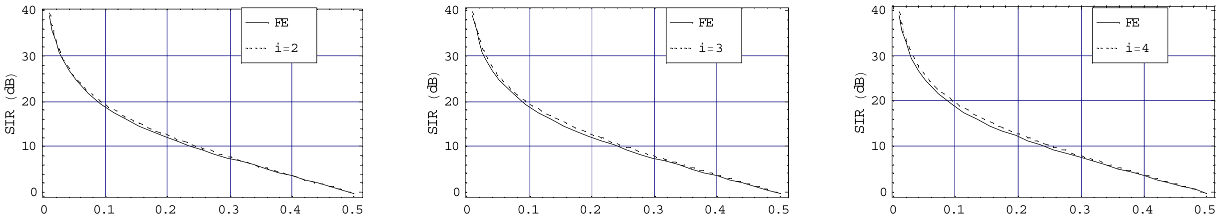
a) b)

Figure 1. ICI power for pulse shape proposed in [15] compared with flipped-exponential FE pulse and polynomial pulse for $\alpha = 1$ and $\alpha = 0.5$ in a 64-subcarrier OFDM system



a) b)

Figure 2. The ICI power (pulse shape proposed in [15]) for different sample location in a 64-subcarrier OFDM system when a) $\alpha = 0.25$; b) $\alpha = 0.35$

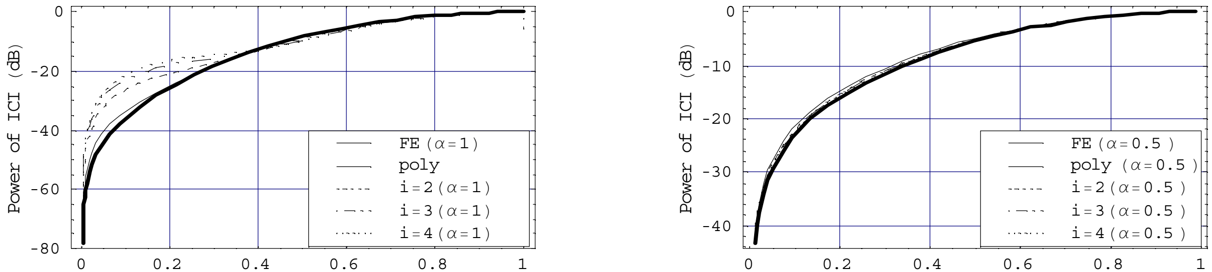


a) b) c)

Figure 3. The SIR for pulse shape proposed in [15] compared with flipped-exponential FE pulse for $\alpha = 0.35$ in a 64-subcarrier OFDM system

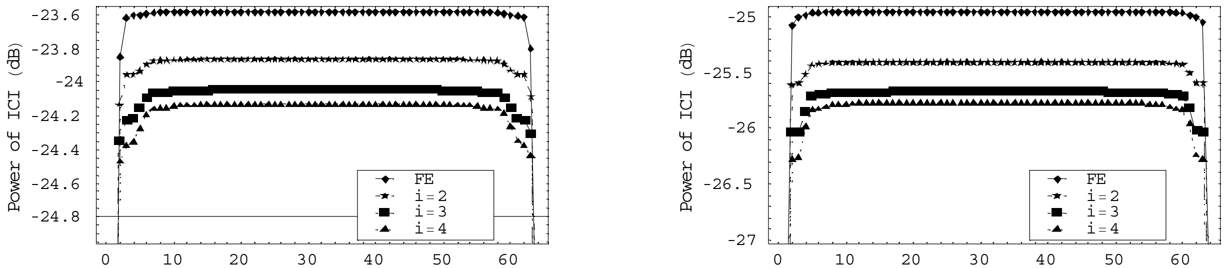
For the rest of the new ISI free Nyquist pulses proposed in [16] and [18] the results obtained are nearly the same with those presented until now for pulses investigated in [15] and are plotted in the Fig. 4 to Fig.7 together with flipped-exponential pulse and polynomial pulses taken as references.

For example in [16] when $\Delta f = 0.51$ then average ICI power is -8.65582dB, -9.08672dB and -9.34421dB for i = 2, i = 3 and i = 4, respectively. The s_4 pulse achieves 1.30105dB and 1.44957dB smaller ICI power than flipped-exponential pulse and polynomial pulse, respectively. The s_2 pulse achieves 0.612666dB and 0.761193dB smaller ICI power than flipped-exponential pulse and polynomial pulse, respectively. The s_3 pulse achieves 1.04356dB and 1.19209dB smaller ICI power than flipped-exponential pulse and polynomial pulse, respectively. (Fig.4a)



a) b)

Figure 4. ICI power for pulse shape propesed in [16] compared with flipped-exponential FE pulse and polynomial pulse for $\alpha = 1$ and $\alpha = 0.5$ in a 64-subcarrier OFDM system



a) b)

Figure 5. The ICI power (pulse shape propesed in [16]) for different sample location in a 64-subcarrier OFDM system when a) $\alpha = 0.25$, b=0.24; b) $\alpha = 0.35$, b=0.34;

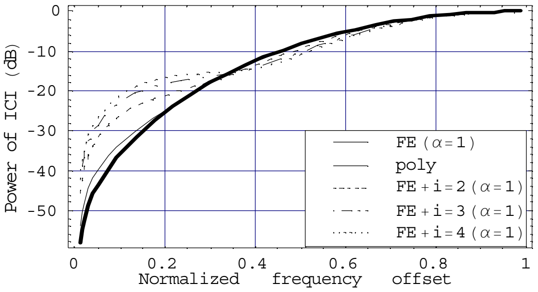


Figure 6. ICI power for pulse shape proposed in [18] compared with flipped-exponential FE pulse and polynomial pulse for $\alpha = 1$ in a 64-subcarrier OFDM system; (the first case proposed).

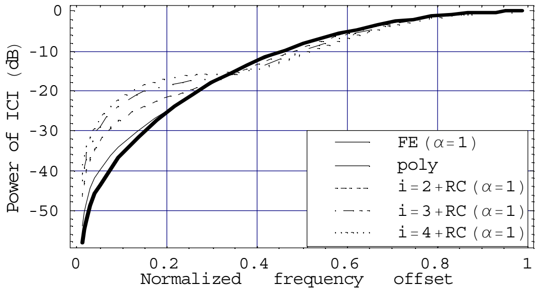


Figure 7. ICI power for pulse shape proposed in [18] compared with flipped-exponential FE pulse and polynomial pulse for $\alpha = 1$ in a 64-subcarrier OFDM system; (the second case proposed).

V. CONCLUSIONS

In this paper we present and evaluate the employment of new *ISI*-free pulses in an OFDM system. The pulses are used for reducing *ICI* in OFDM systems. The results are examined in terms of average *ICI* power and average signal power to average *ICI* power ratio denoted as *SIR*.

When normalized frequency offset is bigger than 0.35 and the roll-off factor $\alpha = 1$ the proposed pulses outperform the references pulses in terms of average ICI power.

The calculations of the equations for the *ICI* power and the *SIR* were performed using *MATHEMATICA*. The aim of doing these simulations was to find out if the new families of proposed pulses [15], [16], [18] show improvement in the reduction of *ICI* caused by the frequency offset in a 64 subcarrier OFDM system. The new pulses appear to be suitable for transmission in OFDM systems and have practical and theoretical importance.

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