A New Family of ISI-Free Pulses Investigated in Terms of ISI and ICI Properties

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Abstract

We proposed a family of new Nyquist pulses based on a composite frequency characteristic and investigated its performances in terms of ISI error probability. The pulses proposed and investigated here are suitable for OFDM use, to reduce the sensitivity of OFDM systems due to frequency offset. The results presented in this paper equal or slightly outperform those of the recently found pulses in terms of inter-carrier interference (ICI) power.
Keywords: *intersymbol-interference (ISI), intercarrier-interference (ICI), frequency offset.*

1 Introduction

We propose a family of new Nyquist pulses that shows comparable or better ISI performance in the presence of sampling errors, as compared with some recently proposed pulses. Usually, Nyquist filters have a low-pass characteristic showing odd-symmetry around cut-off frequency and provide ISI- free transmission.
The most common pulse used in telecommunication is the so-called *raised cosine* (RC) pulse. The tails of the RC Nyquist pulse (for an excess bandwidth, $\alpha > 0$) decay asymptotically as t^{-3}, as it is well known. Several new ISI - free pulses [2],

[3] were proposed that asymptotically decay as t^{-3} and t^{-2}, respectively. They show better performance than RC with respect to timing error sensitivity.

More recently a new family of ISI free and band-limited pulses that can be made to have an ADR (*Asymptotic Decay Rate*)of t^{-k} for any integer value of *k* has been proposed [20]. This family provides flexibility in designing an appropriate pulse even after the roll-off factor has been chosen. It is demonstrated that in terms of sensitivity to timing errors, it is possible to obtain pulses from the new family that perform better than those in [2] and [3]. These pulses were also examined with respect to Inter-Carrier Interference (ICI) in OFDM systems.

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier transmission technique which is widely used in different communication systems. OFDM is adequate for high-speed communication due to its resistance to intersymbol interference (ISI). Intersymbol interference becomes a constraint in high-data-rate communication. OFDM techniques avoid this problem by sending many low speed transmissions simultaneously.

OFDM offers some advantages in higher order modulations and in the networking operation that establish OFDM as the technique of choice for the future wireless networks.

One of the disadvantages of OFDM systems is that it is very sensitive to frequency errors caused by frequency differences between the local oscillators in the transmitter and receiver. A number of methods have been developed to reduce the sensitivity to frequency offset. A pulse shaping technique for the design of pulse shapes for OFDM systems is studied in the following to control and reduce the sensitivity to frequency offset. This is based on combining different types of characteristics. [16], [17].

In this paper we evaluate the performance of new *ISI*-free pulses used in transmission on OFDM systems. The performances are studied with respect to the ISI error probability and ICI in OFDM systems.

2 The construction technique of the new Nyquist pulses

Recent works [2, 3, 4, 5, 20] have reported and examined new families of pulses, which are *ISI*- free. These pulses are produced by a Nyquist low-pass filter with odd symmetry around the ideal cutoff frequency.

In the sequel we illustrate the construction method of the new Nyquist pulses we proposed. The filter characteristic of the new family of pulses is defined for positive frequencies using eq. (1) and is illustrated in figure 1.

The proposed filter frequency characteristic is built combining a constant function, equal with either 0.5, that stretches from $B(1-c)$ to $B(1+c)$, or 1 from 0 to $B(1-\alpha)$, respectively, as well as a $G(f)$ function, as shown in figure 1 for B = 1.

$$H(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ G\left(|f|-B(1-\alpha)\right), & B(1-\alpha)\le|f|\le B(1-c)\\ 1/2, & B(1-c)\le|f|\le B(1+c)\\ 1-G\left(B(1+\alpha)-|f|\right), & B(1+c)\le|f|\le B(1+\alpha)\\ 0, & B(1+\alpha)>|f|\end{cases} \quad (1)$$

where α represents the excess bandwidth; B is the bandwidth corresponding to symbol repetition rate $T=1/2B$ and c is a constant to be determined in order to obtain a minimal value of the error probability.

The function $G(f)$ was chosen in order to satisfy

$$G((1-\alpha)B)=1 \text{ and } G((1-c)B)=1/2 \quad (2)$$

This method is based on the fact, that $H(f)$ was chosen to have a concave shape in the frequency interval $B(1-\alpha)|\le|f||\le B$ in order to transfer some energy to the high frequency spectral range [2], [3], [4].

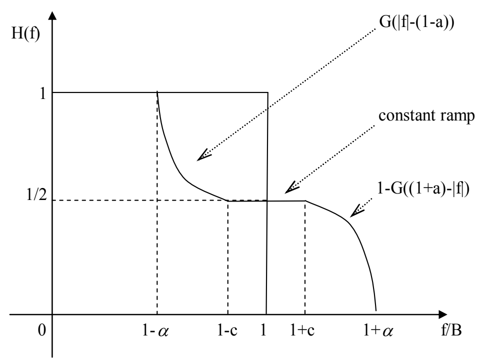


Figure 1 Proposed filter frequency characteristic

The proposed family of pulses examined in this paper equals or slightly outperforms the recently found pulses [2, 3, 4, 5, 20] in terms of ISI and ICI power. In order to illustrate the behavior of the new family of Nyquist pulses reported, we chose for $G(f)$ the polynomial characteristic proposed in [20].

The proposed function $G(f)$ [20] is a n^{th} degree polynomial defined by equation (3):

$$G(f)=\sum_{i=0}^{n} a_i f^{i} \tag{3}$$

The family of ISI free and band-limited polynomial pulses can be made to have an ADR of t^{-k} for any integer value of k. It was demonstrated in [20] that for a 4^{th} degree polynomial $G(f)$ the maximum ADR possible is t^{-4}.

In figure 2 is plotted the odd-symmetry of frequency filter characteristics defined with equation (1) for different value of roll-off factor α. We choose for $G(f)$ a 4^{th} degree polynomial with the ADR of t^{-2}.

The design variables a_2, a_3 and a_4 are those obtained in [20]. The pulse taken as a reference is the polynomial pulse with the parameters set in [20]. Figure 3 illustrates the first two secondary side lobes of the impulse response.

We observe that the first side lobe of the proposed pulses is slightly smaller than the first side lobe of the pulse taken as a reference and the next side lobes follow closely those of the polynomial pulse. This leads to a small decrease of the error probability. Comparing the figures 3a, 3b and 3c we found out that the increase of the excess bandwidth determines a significant decrease of the first side lobe for the reference pulse and the proposed pulses, too.

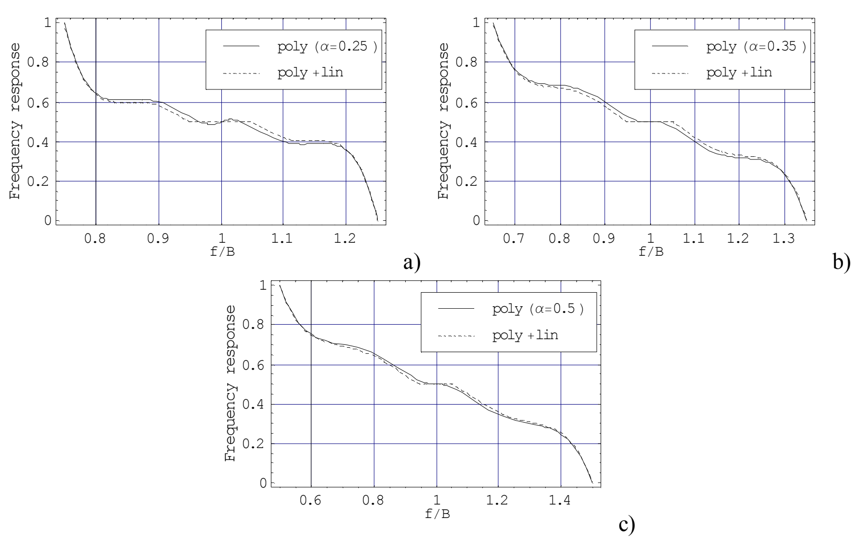


Figure 2 Frequency characteristics of the proposed pulse and the reference pulse (polynomial pulse [20])

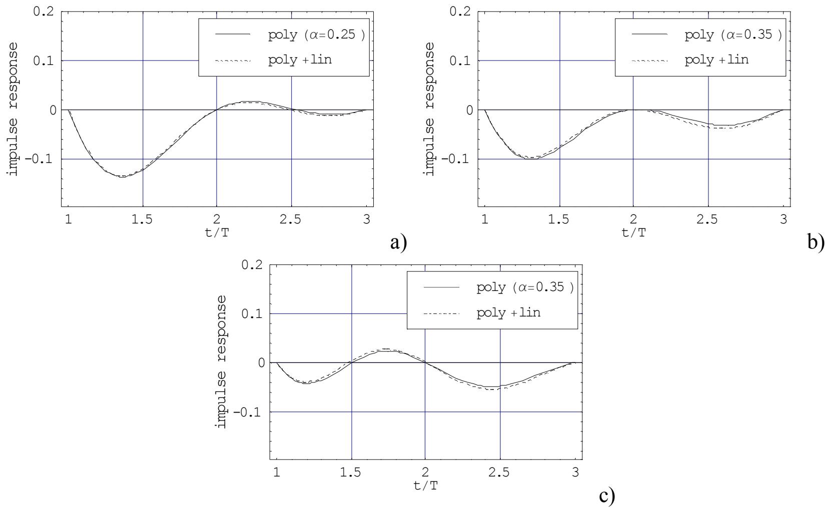


Figure 3 Impulse response of the proposed pulse and the reference pulse (polynomial pulse [20])

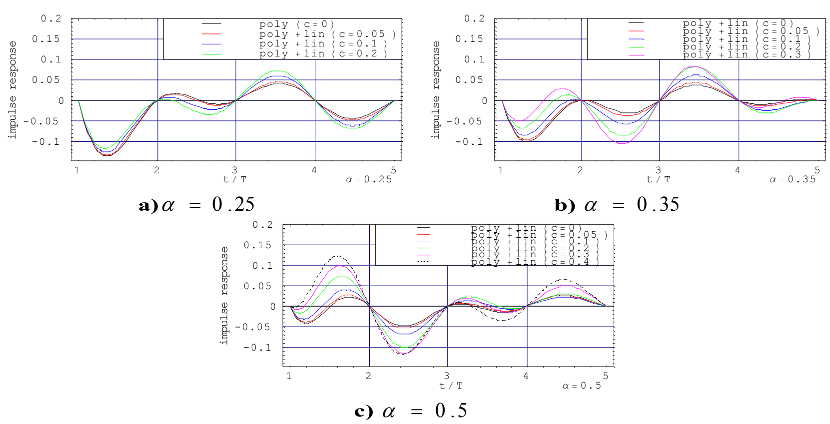


Figure 4 Behavior of the impulse response for different values of the parameter c

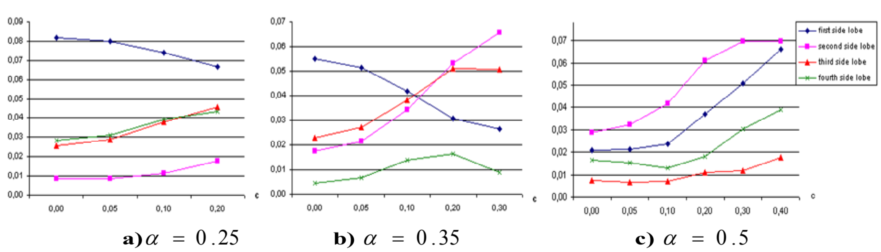


Figure 5 The variation of the firsts fourth side lobes size for different values of the parameter c

In figure 4 are presented the first four side lobes of the impulse response when the parameter *c* is varied on a range from *0* to α . We observe for example, that when $\alpha = 0.25$ and $\alpha = 0.35$ an increasing of *c* factor determine a decrease of the first side lobe with the disadvantage of increasing the next side lobes. The reference pulse is the polynomial pulse [20], which in these figures is marked for the case $c = 0$. The variation of the side lobes size with *c* is presented in figures 5a), 5b) and 5c) for different values of the roll-off parameter α . From figure 5c) we observe that for $\alpha = 0.5$, an increase of *c* determines a rise of the side lobes. At a closer look we notice that the first side lobe is smaller than the second side lobe for any value of *c.*

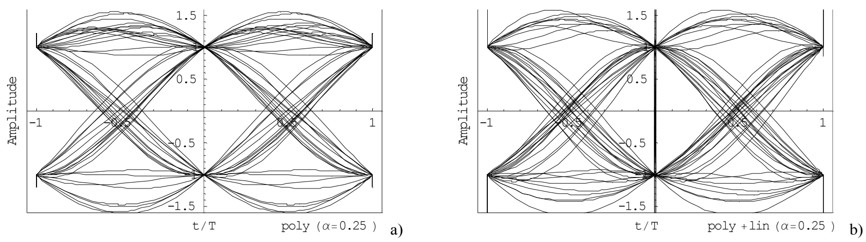


Figure 6 Eye diagram of the pulse taken as reference [20] and the new pulse proposed $\alpha = 0.25$

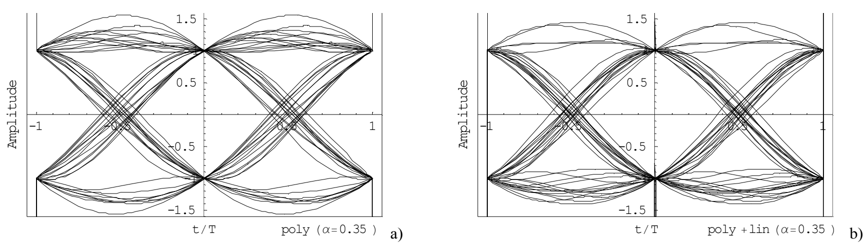


Figure 7 Eye diagram of the pulse taken as reference [20] and the proposed pulse $\alpha = 0.35$

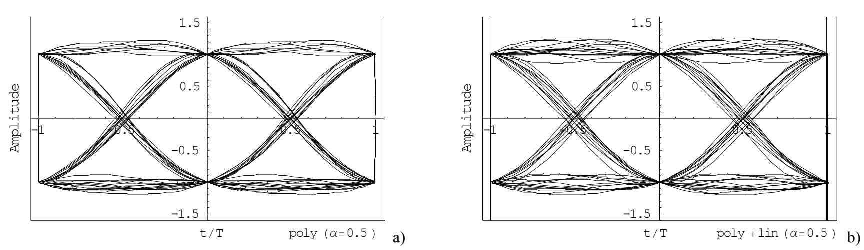


Figure 8 Eye diagram of the pulse taken as reference [20] and the proposed pulse $\alpha = 0.5$

3 Simulation results

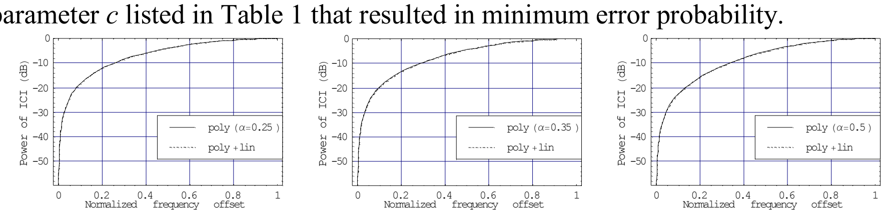
Figures 6, 7 and 8 illustrate the receiver eye diagram for the new pulses. The effects of the ICI are estimated in the following.
When the receiver eye diagram is sampled off center, as in practical receivers, timing error results in an increase of the average symbol error probability [2, 3]. The new families of Nyquist pulses show reduced maximum distortion, a more open receiver eye and decreased symbol error probability [4] in the presence of timing error, as compared with the recent pulses reported [2, 3, 4, 5] with the same roll-off factor.
The ISI error probability is calculated using the method of [12] for the proposed family of pulses and is illustrated in Table 1, together with those for the polynomial pulse. The results are computed using $T_f = 40$ and $M = 61$ for $N=2^{10}$ interfering symbols and SNR = 15 dB.
From the results listed in Table 1, we observe that the new pulses outperform the polynomial pulse. As expected, the error probability is reduced if the roll-off factor is increased. The parameter c encompasses the range (0.01 – 0.09), its optimal value in terms of error probability depending on the excess bandwidth α and the timing error.
The decrease of the first side lobe induces a more open eye in the eye diagram. As observed from Table 1, sampling the signal with a different offset with respect to ideal zero ISI moments determines an increase of the error probability.

Table 1: Error probability of the proposed Nyquist pulses for $N=2^{10}$ interfering symbols and SNR = 15 dB

α	pulse	a_2	a_3	a_4	t/T=0.05		t/T=0.1		t/T=0.2	
0.25	poly	40	-100	85	**$4.734*10^{-8}$**		**$8.834*10^{-7}$**		**$2.241*10^{-4}$**	
	new				c=0.05	$4.729*10^{-8}$	c=0.02	$8.804*10^{-7}$	c=0.02	$2.220*10^{-4}$
0.35	poly	31	-80	69	**$3.290*10^{-8}$**		**$3.839*10^{-7}$**		**$6.563*10^{-5}$**	
	new				c=0.08	$3.228*10^{-8}$	c=0.05	$3.807*10^{-7}$	c=0.02	$6.561*10^{-5}$
0.5	poly	25	-64	55	**$2.057*10^{-8}$**		**$1.354*10^{-7}$**		**$1.520*10^{-5}$**	
	new				c=0.09	$2.032*10^{-8}$	c=0.05	$1.348*10^{-7}$	c=0.01	$1.519*10^{-5}$

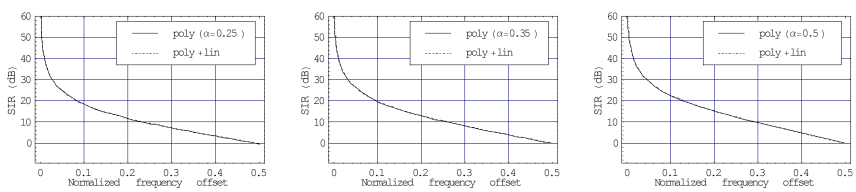
The error probability measures the performances of the pulses regarding ISI and includes the effects of noise, synchronization error and distortion.
The proposed pulses can also be used for reducing average *ICI* power in OFDM systems [19]. In the sequel we followed the same model as in [19] in order to evaluate the average *ICI* power and the average signal power to average *ICI* power ratio (*SIR*). The simulations results are obtained for a 64-subcarrier OFDM system.

In figure 9 it is plotted the average ICI power for the proposed pulses together with the *4th* degree polynomial pulse taken as reference. The ICI power for the new pulses has almost the same values as the ICI power for polynomial pulse.
The increase of the roll-off factor α is expected to result in the reduction of *ICI* power. An increase of α results in reducing the side lobes size, observed in the impulse response. In figure 11 it is plotted and compared the average *ICI* power of the new pulse proposed when the excess bandwidth α takes different values.
When we concern on the comparative performance of this pulses in a 64-subcarrier OFDM system [19] in terms of the average signal power to average *ICI* power ratio, denoted as *SIR*, the results are plotted in figures 10 a, 10 b and 10 c, respectively.
Figure 12 compares the SIR of the new proposed pulse for different value of the roll-off factor α. As expected the SIR ratio is increasing when the excess bandwidth is larger. The average ICI power and SIR ratio are plotted as functions of the normalized frequency offset, ΔfT. In figures 9-12 we chose the value of parameter *c* listed in Table 1 that resulted in minimum error probability.



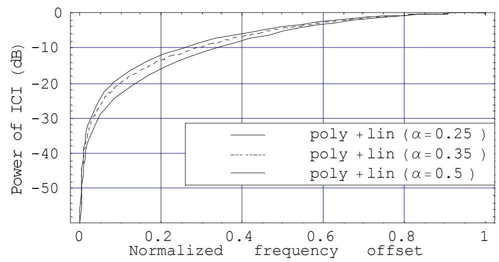
a) *c=0.05* **b)** *c=0.08* **c)** *c=0.09*

Figure 9 ICI power for new pulse shape proposed compared with *poly* pulse [20] in a 64-subcarrier OFDM system



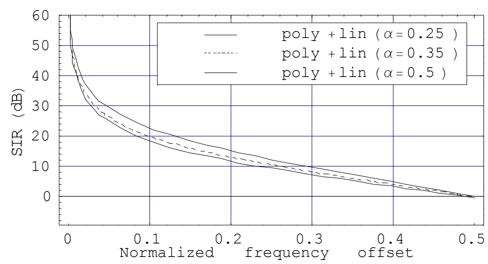
a) *c=0.05* **b)** *c=0.08* **c)** *c=0.09*

Figure 10 SIR for pulse shape proposed compared with *poly* pulse [20] in a 64-subcarrier OFDM system



$\alpha = 0.25$ $c=0.05;$ $\alpha = 0.35$ $c=0.08;$ $\alpha = 0.5$ $c=0.09$

Figure 11 ICI power for the proposed pulse shape in a 64-subcarrier OFDM system



$\alpha = 0.25$ $c=0.05;$ $\alpha = 0.35$ $c=0.08;$ $\alpha = 0.5$ $c=0.09$

Figure 12 SIR for the proposed pulse shape in a 64-subcarrier OFDM system

4 Conclusions

We proposed and investigated the performances of a family of new improved Nyquist pulses based on a composite frequency characteristic that show a decreased symbol error probability in the presence of timing errors, as compared with the *4^th^* degree polynomial pulse with the same roll-off factor and design variables a_2, a_3 and a_4 set in [20].

We also presented and evaluated the use of these new *ISI*-free pulses in an OFDM system in order to reduce *ICI*. The results are examined in terms of average *ICI* power and average signal power to average *ICI* power ratio denoted as *SIR*.

The new pulses show improvement in the reduction of *ICI* caused by the frequency offset and appear to be suitable for transmission in OFDM systems.

The calculations of error probability, *ICI* power and *SIR* were done using *MATHEMATICA*.

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