IMPROVED NYQUIST FILTER CHARACTERISTIC USING SPLINE FUNCTIONS INTERPOLATION

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ABSTRACT

We propose a family of new Nyquist pulses that shows comparable or better ISI performance in the presence of sampling errors, as compared with some recently proposed pulses. Several new ISI free pulses [2], [3] were proposed that asymptotically decay respectively as t^{-3} and as t^{-2}. More recently a new family of ISI free and band-limited pulses that can be made to have an ADR of t^{-k} for any integer value of k has been proposed [4]. This family provides flexibility in designing an appropriate pulse even after the roll-off factor has been chosen. The performances of new *ISI*-free pulses are studied with respect to the ISI error probability and inter-carrier interference (ICI) in OFDM systems.

1. INTRODUCTION

We proposed to study the behavior of the low-pass filter with odd symmetry about the corresponding ideally band-limited cut-off frequency that is defined for positive frequencies. Recent works [2, 3, 4] have reported and examined new families of pulses which are inter-symbol interference (*ISI*)- free. These pulses are produced by a Nyquist low-pass filter with odd symmetry around the ideal cutoff frequency.

The filter characteristic of the new family of pulses is defined for positive frequencies using equation (1) and is illustrated in figure 1.

$$X(f)=\begin{cases} 1, & |f|\le(1-\alpha)B \\ a_1, & (1-\alpha)B\le|f|\le\left(1-\frac{2k-1}{2k+1}\alpha\right)B \\ \dots & \\ a_k, & \left(1-\frac{3}{2k+1}\alpha\right)B\le|f|\le\left(1-\frac{1}{2k+1}\alpha\right)B \\ 1/2, & \left(1-\frac{1}{2k+1}\alpha\right)B\le|f|\le\left(1+\frac{1}{2k+1}\alpha\right)B \\ 1-a_k, & \\ \dots & \\ 1-a_1, & \left(1+\frac{2k-1}{2k+1}\alpha\right)B\le|f|\le(1+\alpha)B \\ 0, & |f|>(1+\alpha)B \end{cases} \quad (1)$$

The characteristic is built combining rectangular functions, as shown in figure 1.

The transfer function presents $2k+2$ intervals where is constant over frequency intervals of equal width [7].

The characteristic is built so, that the shaded surfaces obeying odd-symmetry in figure 1 are equal.

The rectangular functions used for this example appear to be desirable in minimizing the bandwidth and the results evaluated in terms of error probability for a large number of parameters outperform those in most recent works.

The function $X(f)$ was chosen in order to satisfy:

$$X((1-\alpha)B)=1 \text{ and } X(B)=1/2 \quad (2)$$

Figure 1 illustrates the filter frequency characteristic defined by eq. (1) for positive frequencies and will be called further in this paper *stair-shape characteristic*.

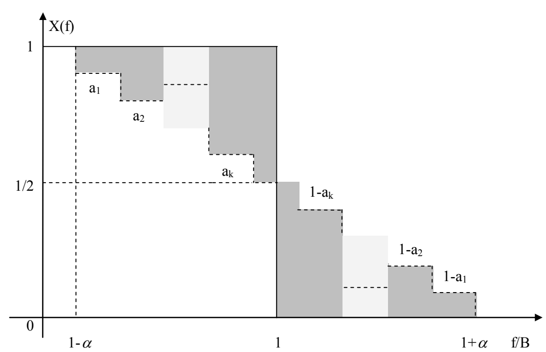


Fig. 1 Proposed filter frequency characteristic

As the rectangular filter characteristic cannot be physically implemented, we illustrate in the sequel the construction method of the new Nyquist pulse to replace the ideal characteristics, making an approximation by a polynomial, using interpolation by spline functions.

The spline approximation is a piece-wise polynomial approximation [8].

From the spline functions we have selected so far the cubic spline functions, which are the most important ones from a practical point of view.

They are smooth functions and when are used for interpolation, they do not have the oscillatory behavior that characterizes high degree polynomial interpolation. Cubic spline functions are simple to use and calculate.

2. FILTER DESIGN

In the sequel, we study the behaviour of the proposed pulses produced by the low-pass filter defined with relation (1), in terms of the inter-symbol interference for two particular cases.

2.1 Design examples

Case 1.

In the first case we consider k = 1 and the *stair-shape* characteristic is defined by the following equation:

$$X_1(f)=\begin{cases}1, & |f|\le(1-\alpha)B\\ a_{1,} & (1-\alpha)B\le|f|\le\left(1-\dfrac{1}{3}\alpha\right)B\\ 1/2, & \left(1-\dfrac{1}{3}\alpha\right)B\le|f|\le\left(1+\dfrac{1}{3}\alpha\right)B\\ 1-a_1 & \left(1+\dfrac{1}{3}\alpha\right)B\le|f|\le(1+\alpha)B\\ 0, & (1+\alpha)B>|f|\end{cases} \quad (3)$$

The function $X_1(f)$ is constant on two intervals in the range $[0,(1-\alpha)B]$ and around B. Its precise shape is determined by the parameters α and a_1. The spline function associated to the $X_1(f)$ function passes through the points given in Table 1.

Figure 2 illustrates the spline filter characteristics for different values of roll-off factor α together with the stair shape characteristic, displaying the points listed in Table 1.

In figure 3 we have the spline filter characteristics compared with the polynomial function.

The impulse response is represented in figure 4 for different values of the roll-off factor α and the parameter a_1.

An increase of the roll-off factor α determines a decrease of the first side lobes in time domain.

Table 1. Interpolation points for the spline function

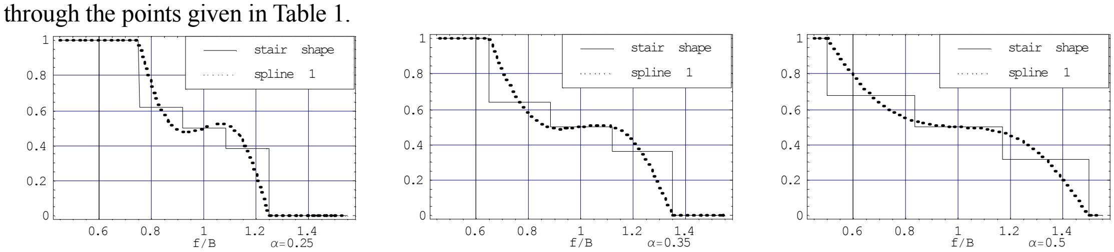
f	$1-\alpha$	$1-2\alpha/3$	1	$1+2\alpha/3$	$1+\alpha$
$S_1(f)$ spline 1	1	a_1	1/2	$1-a_1$	0

Case 2.

In this case we consider k = 2 and the resulting *stair-shape* characteristic is obtained using relation (4).

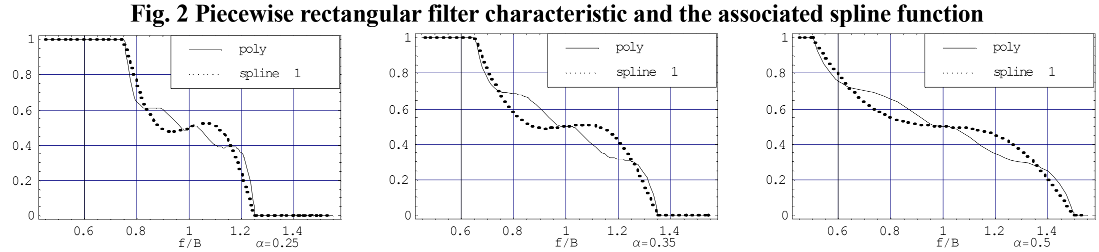
We observe that the function $X_2(f)$ is constant over five intervals in the range $[(1-\alpha)B,(1+\alpha)B]$ and is built using the parameters a_1 and a_2.

The spline function associated to the $X_2(f)$ function passes through the points given in Table 2.



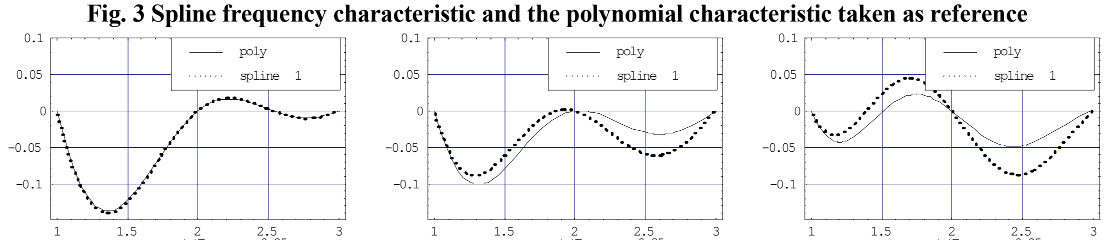
a) b) c)

Fig. 2 Piecewise rectangular filter characteristic and the associated spline function



a) b) c)

Fig. 3 Spline frequency characteristic and the polynomial characteristic taken as reference

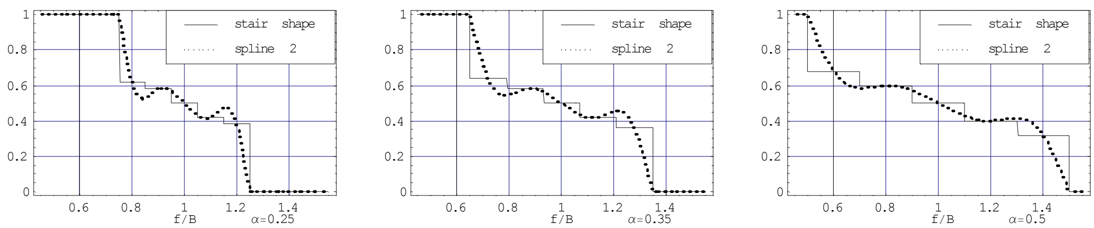


a) b) c)

Fig. 4 First side lobs of the impulse response for the spline 1 pulse shape and polynomial pulse

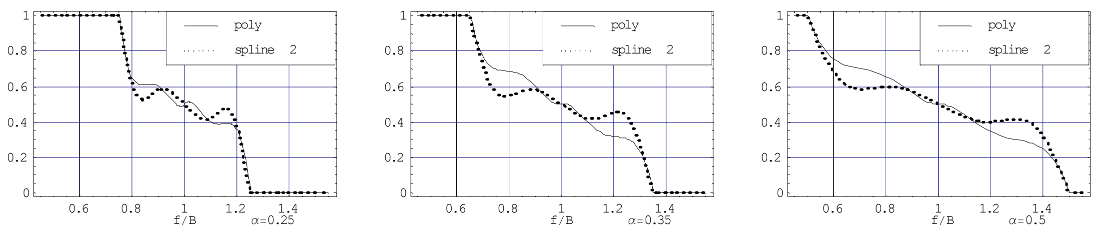
Table 2 Interpolation points for the spline 2 function

f	$1-\alpha$	$1-4\alpha/5$	$1-2\alpha/5$	1	$1+2\alpha/5$	$1+4\alpha/5$	$1+\alpha$
$S_2(f)$ spline 2	1	$a_{1,}$	$a_{2,}$	1/2	$1-a_{2,}$	$1-a_{1,}$	0



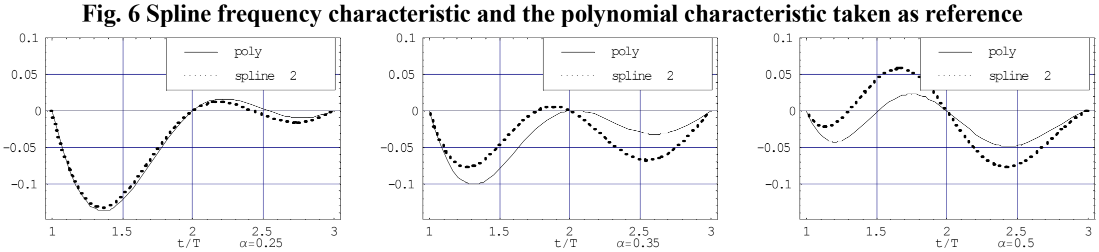
a) b) c)

Fig. 5 Piecewise rectangular filter characteristic and the associated spline function



a) b) c)

Fig. 6 Spline frequency characteristic and the polynomial characteristic taken as reference



a) b) c)

Fig. 7 First side lobs of the impulse response for the spline 2 pulse shape and polynomial pulse

$$X_2(f)=\begin{cases}1, & |f|\le(1-\alpha)B\\ a_1, & (1-\alpha)B\le|f|\le\left(1-\frac{3}{5}\alpha\right)B\\ a_2 & \left(1-\frac{3}{5}\alpha\right)B\le|f|\le\left(1-\frac{1}{5}\alpha\right)B\\ 1/2, & \left(1-\frac{1}{5}\alpha\right)B\le|f|\le\left(1+\frac{1}{5}\alpha\right)B\\ 1-a_2, & \left(1+\frac{1}{5}\alpha\right)B\le|f|\le\left(1+\frac{3}{5}\alpha\right)B\\ 1-a_{1,} & \left(1+\frac{3}{5}\alpha\right)B\le|f|\le(1+\alpha)B\\ 0, & |f|>(1+\alpha)B\end{cases}\qquad(4)$$

Figure 5 illustrates the spline filter characteristics for different values of roll-off factor α together with the *stair-shape* characteristic displaying the points listed in Table 2. In figure 6 we have the spline filter characteristics compared with the polynomial function. The impulse response is represented in figure 4 for different values of the roll-off factor α and the parameters a_1 and a_2. An increase of the roll-off factor determines a decrease of the first side lobes in time domain, as in the previous case.

3. SIMULATION RESULTS

3.1 Error Probability

The error probability measures the performances of the pulses regarding inter-symbol interferences and includes the effects of noise, synchronization error and distortion. The ISI error probability P_e was evaluated as in [6] using Fourier series, with $T_f = 40$ and $M = 61$ for N=2^{10} interfering symbols and SNR=15 dB, in the presence of time sampling errors for pulses produced by $X_1(f)$ for different values of parameter a_1 and roll-off factor α for all the cases. In Table 3 are listed the minimum values found for error probability for roll-off factor $\alpha = 0.25, 0.35, \text{and } 0.5$ and timing offset $t/T_B = 0.05, 0.1, \text{and } 0.2$. For the associated spline function we used the values of parameter a_1 found for minimum error probability obtained with the pulses produced by $X_1(f)$, while in case 2 we introduced a new parameter a_2 and used the same calculation method for P_e.

Table 3 ISI error probability of the proposed Nyquist pulses for $N = 2^{10}$ interfering symbols and SNR = 15 dB

Stair shape X_1	$t/T_B = 0.05$		$t/T_B = 0.1$		$t/T_B = 0.2$	
Spline 1 / poly	a_1	P_e	a_1	P_e	a_1	P_e
$\alpha = 0.25$	0.62	$4.55572*10^{-8}$	0.64	$8.42322*10^{-7}$	0.66	$2.13259*10^{-4}$
		$5.13585*10^{-7}$		$1.0612*10^{-6}$		$2.81033*10^{-4}$
	40 -100 85	$4.734*10^{-8}$	40 -100 85	$8.834*10^{-7}$	40 -100 85	$2.241*10^{-4}$
$\alpha = 0.35$	0.64	$3.09292*10^{-8}$	0.68	$3.78826*10^{-7}$	0.7	$7.36233*10^{-5}$
		$3.50207*10^{-8}$		$4.57625*10^{-7}$		$8.63592*10^{-5}$
	31 -80 69	$3.290*10^{-8}$	31 -80 69	$3.839*10^{-7}$	31 -80 69	$6.563*10^{-5}$
$\alpha = 0.5$	0.68	$2.0085*10^{-8}$	0.68	$1.44*10^{-7}$	0.72	$2.32392*10^{-5}$
		$2.19405*10^{-8}$		$1.72002*10^{-7}$		$2.11773*10^{-5}$
	25 -64 55	$2.057*10^{-8}$	25 -64 55	$1.354*10^{-7}$	25 -64 55	$1.520*10^{-5}$

Table 4 ISI error probability of the proposed Nyquist pulses for $N = 2^{10}$ interfering symbols and SNR = 15 dB

Stair shape X_2	$t/T_B = 0.05$			$t/T_B = 0.1$			$t/T_B = 0.2$		
Spline 2 / poly	a_1	a_2	P_e	a_1	a_2	P_e	a_1	a_2	P_e
$\alpha = 0.25$	0.62	0.58	$4.53042*10^{-8}$	0.64	0.58	$8.26096*10^{-7}$	0.66	0.6	$2.03858*10^{-4}$
			$4.8703*10^{-8}$			$9.62596*10^{-7}$			$2.46822*10^{-4}$
	40 -100 85		$4.734*10^{-8}$	40 -100 85		$8.834*10^{-7}$	40 -100 85		$2.241*10^{-4}$
$\alpha = 0.35$	0.64	0.58	$3.05974*10^{-8}$	0.68	0.6	$3.56887*10^{-7}$	0.7	0.62	$6.4535*10^{-5}$
			$3.31799*10^{-8}$			$4.15843*10^{-7}$			$7.68802*10^{-5}$
	31 -80 69		$3.290*10^{-8}$	31 -80 69		$3.839*10^{-7}$	31 -80 69		$6.563*10^{-5}$
$\alpha = 0.5$	0.68	0.6	$1.9494*10^{-8}$	0.68	0.6	$1.33245*10^{-7}$	0.72	0.62	$1.8073*10^{-5}$
			$2.08965*10^{-8}$			$1.57947*10^{-8}$			$2.05854*10^{-5}$
	25 -64 55		$2.057*10^{-8}$	25 -64 55		$1.354*10^{-7}$	25 -64 55		$1.520*10^{-5}$

4. CONCLUSIONS

Better results for error probability can be obtained when we introduce a new level in the stair shape characteristic, for sampling moment $t/T_B = 0.2$, as inferred from Table 3 and Table 4.

The results for error probability are comparable with the *4th* degree polynomial pulse with the same roll-off factor [4].

As a further work we are going to extend spline approximation to a larger number of points for interpolation.

The pulses obtained and investigated are suitable for OFDM use, to reduce the sensitivity of OFDM systems due to frequency offset.

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