A CLASS OF IMPROVED PULSES GENERATED BY NYQUIST FILTERS

Nicolae Dumitru ALEXANDRU
Gh. Asachi" Technical University of Iaşi
Department of Telecommunications
Bd. Carol I no 11, 700506 - Iaşi, ROMANIA
nalex@etc.tuiasi.ro

Maria Liliana ALEXANDRU
"G. Ibrăileanu" Highschool Iaşi
l_alex21@yahoo.com

Ligia Alexandra ONOFREI
"Ştefan cel Mare" University of Suceava
str. Universităţii nr.13, RO-720225 Suceava
onofreial@eed.usv.ro

Abstract: *A novel class of ISI-free pluses is presented and investigated. We propose and investigate a class of new Nyquist pulses that shows comparable or better ISI performance in the presence of sampling errors, as compared with some recently proposed pulses.*
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1.Introduction

Recently, improved Nyquist pulses that show smaller maximum distortion, more open receiver eye and a smaller symbol error rate in the presence of symbol timing error were reported [2,3 and 4]. They are defined by

$$S_i(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ G\big((|f|-B(1-\alpha)\big), & B(1-\alpha)|\le|f||\le B\\ 1-G\big((B(1+\alpha)-|f|\big), & B<|f|\le B(1+\alpha)\\ 0, & B(1+\alpha)<|f|\end{cases} \quad (1)$$

where $G(f)$ is a function satisfying $G(0)=1$. In [2,3 and 4] $G(f)$ was chosen to have a particular shape in the frequency interval $B(1-\alpha)|\le|f||\le B$ in order to transfer some energy to the high frequency spectral range. This results in a pulse that decays asymptotically as t^{-2} as compared with t^{-3} for the RC pulse, but with the advantage that the eye diagram is more open and, as a consequence, a better bit error rate is obtained.

Two recent contributions showed that improved Nyquist pulses could be obtained with the *flipped-*$G(f)$ technique, e.g. *flipped-exponential* [2] and *flipped-hyperbolic secant* or *flipped-inverse hyperbolic secant* [3].
The envelope of the impulse response decays as t^{-2} or t^{-3} at best, since the functions and their flipped counterparts are continuous at $f_n=1$.
The first derivative of the *flipped-hyperbolic secant* is continuous at $f_n=1$, which accounts for its steeper decay. The *flipped-exponential* technique uses $G(f)=e^{f}$ and $\beta=\ln 2/(\alpha B)$, while in [3] $G(f)=\sec h(f)$ and $\beta=\gamma=\ln(\sqrt{3}+2)/(\alpha B)$ or $X(f)=1-\frac{1}{2\alpha B\gamma}arc\sec h(f)$ with $\beta=\frac{1}{2\alpha B}$.

2. A class of new Nyquist pulses

We propose o class of new Nyquist pulses with piece-wise characteristics, that are defined for positive frequencies and even *i* as

$$S_i(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ G\big((|f|-B(1-\alpha)\big), & B(1-\alpha)\big|\le\big|f\big|\big|\le B(1-b)\\ H_i\big(|f|-B\big), & B(1-b)\big|\le\big|f\big|\big|\le B\\ 1-H_i\big(|f|-B\big), & B<|f|\le B(1+b)\\ 1-G\big((B(1+\alpha)-|f|\big), & B(1+b)<|f|\le B(1+\alpha)\\ 0, & B(1+\alpha)<|f|\end{cases} \qquad (2)$$

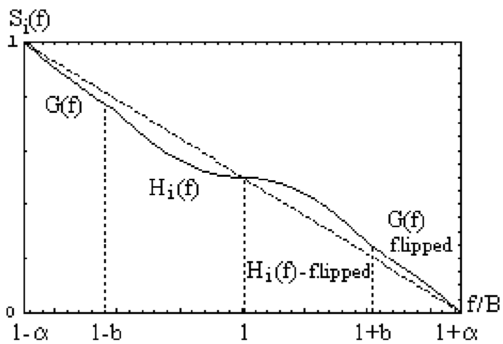

Figure 1 Proposed filter characteristic

The Nyquist filter characteristic is obtained from combining two types of characteristics with odd-symmetry. Here $G(f)$ is the *flipped-exponential* characteristic proposed in [2] and $H_i(f)$ is the family of parabolic, cubic and quartic ramps proposed in [5].

$$G(f)=e^{f}$$

$$H_i(f)=-(1)^i\,\frac{(f-B)^i}{B^i b^i}\left(e^{\frac{\ln 2(b-\alpha)}{\alpha}}-\frac{1}{2}\right)+\frac{1}{2} \qquad (2)$$

For *i* odd they show odd symmetry around *B* and their definition is

$$S_i(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ G(f), & B(1-\alpha)\big|\le\big|f\big|\big|\le B(1+b)\\ H_i(f) & B(1-b)\big|\le\big|f\big|\big|\le B(1+b)\\ G(f)-\mathit{flipped} & B(1+b)\big|\le\big|f\big|\big|\le B(1+\alpha)\\ 0, & B(1+\alpha)<|f|\end{cases} \qquad (3)$$

For *i* even, the vestigial symmetry is obtained by choosing $H_i(f)$ for the frequency interval $B(1-\alpha)\le f\le B$ and $1-H_i(f)$ for $B\le f\le B(1+\alpha)$.

The expressionss were derived imposing conitnuity conditions at $f=B(1-b)$ and $f=B(1+b)$ and a value of 0.5 at $f=B$.

This technique will be denoted in the sequel as *double-ramp flipped-*$G(f)$ and is illustrated in Fig.1.

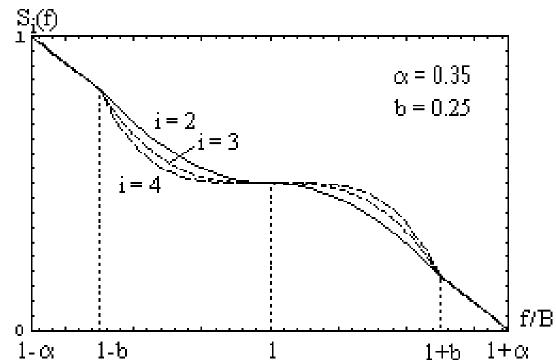

Figure 2 A class of proposed frequency characteristics (positive frequencies)

Figure 2 illustrates this class of new Nyquist filter characteristics for $i=2,3,\mathit{and}\ 4$.

Since they are more concave than the FE pulse, a decrease of the first side lobe in time domain is to be expected, as shown in Fig.3, where a time-scaled replica of pulses is represented for $\alpha=0.25$. The impulse responses $s_i(t)$ are given in the Appendix A.

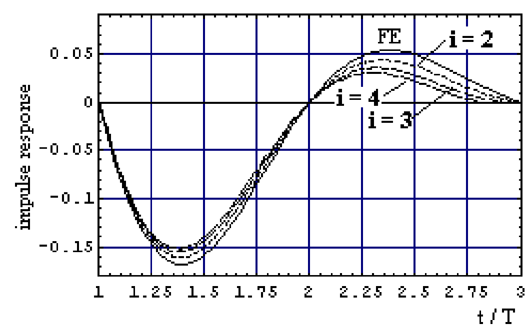

Figure 3 Impulse responses $\alpha=0.25$

Their behavior around $t/T = 3,4,\cdots$ is more flat, which accounts for their better properties regarding the error probability when sampled with a small time offset.

A look at Fig.3 that further illustrates the decay of impulse responses reveals that the new pulse defined by (1) and (2) with $i = 2$ follows closely the FE pulse.

Regarding the other pulses (i = 3 *and* 4), though the decrease of the first side lobe is more significant, the side lobes are significantly larger, which results in increased ISI. The behaviour is similar to that of FE pulse where increasing α results in decreased error probability.

3. Error probability

When the receiver eye is sampled off center, as in practical receivers, timing error results in an increase of the average symbol error probability [2, 3, 14].

This is calculated using the method of [13] for all proposed pulses and illustrated in Table I, together with those for FE pulse.

3. Conclusions

A new class of Nyquist pulses that show reduced maximum distortion, a more open receiver eye and decreased symbol error probability in the presence of timing error as compared with the FE pulse [2] with the same roll-off factor was introduced. Its transmission properties were thoroughly investigated and show that the pulses have practical importance.

Table 1: ISI error probability of several Nyquist pulses for N=2^{10} interfering symbols and SNR = 15 dB

P_e	B=1		$t/T_B = \pm 0.05$	$t/T_B = \pm 0.1$	$t/T_B = \pm 0.2$	$t/T_B = \pm 0.3$
	α	b				
FE	0.25		$5.81166*10^{-8}$	$1.29804*10^{-6}$	$3.56785*10^{-4}$	$1.45241*10^{-2}$
	0.35		$3.92526*10^{-8}$	$5.40211*10^{-7}$	$1.01287*10^{-4}$	$5.88798*10^{-3}$
	0.5		$2.41342*10^{-8}$	$1.85795*10^{-7}$	$2.08778*10^{-5}$	$1.57719*10^{-3}$
c2	0.25	0.24	$5.35869*10^{-8}$	$1.1125*10^{-6}$	$2.95331*10^{-4}$	$1.25179*10^{-2}$
	0.35	0.34	$3.59187*10^{-8}$	$4.69738*10^{-7}$	$8.91204*10^{-5}$	$5.17815*10^{-3}$
	0.5	0.49	$2.22421*10^{-8}$	$1.69435*10^{-7}$	$2.27295*10^{-5}$	$1.8209*10^{-3}$
c3	0.25	0.24	$5.14109*10^{-8}$	$1.05201*10^{-6}$	$2.82402*10^{-4}$	$1.1828*10^{-2}$
	0.35	0.34	$3.46657*10^{-8}$	$4.65864*10^{-7}$	$9.6416*10^{-5}$	$5.39191*10^{-3}$
	0.5	0.49	$2.19888*10^{-8}$	$1.77572*10^{-7}$	$3.03206*10^{-5}$	$2.64191*10^{-3}$
c4	0.25	0.24	$5.06443*10^{-8}$	$1.04682*10^{-6}$	$2.87321*10^{-4}$	$1.17539*10^{-2}$
	0.35	0.34	$3.44572*10^{-8}$	$4.8368*10^{-7}$	$1.08168*10^{-4}$	$5.83255*10^{-3}$
	0.5	0.49	$2.23808*10^{-8}$	$1.91963*10^{-7}$	$3.92872*10^{-4}$	$3.54274*10^{-3}$

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Appendix

$$
\begin{aligned}
c_2(t) = \frac{1}{8}\Bigg\{ & -\frac{8\sin((\alpha-1)B\pi t)}{\pi t} + \\
& \frac{4aB\left(\cos((\alpha-1)B\pi t)\log(4) + 2aB\pi t\sin((\alpha-1)B\pi t) - 2^{\frac{b}{\alpha}}\left(\cos((b-1)B\pi t)\log(2) + \alpha B\pi t\sin((b-1)B\pi t)\right)\right)}{\alpha^2 B^2\pi^2 t^2 + \log(2)^2} + \\
& \frac{1}{b^2B^2\pi^3t^3}\left(8\left(\left(1-2^{\frac{b}{\alpha}}\right)\left(\sin(B\pi t)+\sin((-1+b)B\pi t)\right) + bB\pi t\left(\left(2^{\frac{b}{\alpha}}-1\right)\cos((b-1)B\pi t) + \frac{1}{2}bB\pi t\left(\sin(B\pi t) + 2^{\frac{b}{\alpha}}\sin((b-1)B\pi t)\right)\right)\right)\right) + \\
& \frac{1}{b^2B^2\pi^3t^3}\left(8\left(\left(2^{\frac{b}{\alpha}}-1\right)\left(-\sin(B\pi t)+\sin((1+b)B\pi t)\right) + bB\pi t\left(\left(1-2^{\frac{b}{\alpha}}\right)\cos((1+b)B\pi t) - \frac{1}{2}bB\pi t\left(\sin(B\pi t) + \left(2^{\frac{b}{\alpha}}-2\right)\sin((1+b)B\pi t)\right)\right)\right)\right) + \\
& 4\Bigg(\frac{2\left(\sin((1+\alpha)B\pi t) - \sin((1+b)B\pi t)\right)}{\pi t} + \\
& \frac{aB\left(-2\cos((1+\alpha)B\pi t)\log(2) - 2\alpha B\pi t\sin((1+\alpha)B\pi t) + 2^{\frac{b}{\alpha}}\left(\cos((1+b)B\pi t)\log(2) + \alpha B\pi t\sin((1+b)B\pi t)\right)\right)}{\alpha^2B^2\pi^2t^2 + \log(2)^2}\Bigg)\Bigg\}
\end{aligned}
$$

$$c_3(t)=\frac{1}{16B^3}\left\{\cos((-1+b)B\pi t)\left(\frac{16\left(3-3\cdot 2^{\frac{b}{\alpha}}\right)}{b^3\pi^4t^4}+\frac{24\left(2^{\frac{b}{\alpha}}-1\right)B^2}{b\pi^2t^2}-\frac{2^{3+\frac{b}{\alpha}}\alpha B^4\log(2)}{\alpha^2B^2\pi^2t^2+\log(2)^2}\right)+\right.$$

$$\frac{1}{b^3\pi^4t^4\left(\alpha^2B^2\pi^2t^2+\log(2)^2\right)}\left\{16\left[\cos((1+b)B\pi t)\left(-3\left(-1+2^{\frac{b}{\alpha}}\right)\alpha^2B^2\pi^2t^2\left(\frac{b^2B^2\pi^2t^2}{2}-1\right)+\right.\right.\right.$$

$$\left.2^{-1+\frac{b}{\alpha}}\alpha b^3B^4\pi^4t^4\log(2)-3\left(-1+2^{\frac{b}{\alpha}}\right)\left(-1+\alpha^2B^2\pi^2t^2\right)\log(2)^2\right)+$$

$$bB\pi t\left(2b^2B^2\pi^2t^2\cos(\alpha B\pi t)\log(2)^2\sin(B\pi t)+\alpha b^2B^3\pi^3t^3\log(4)\sin(B\pi t)\sin(\alpha B\pi t)+\right.$$

$$\left.\left.\left.\left.\left(3\left(2^{\frac{b}{\alpha}}-1\right)\alpha^2B^2\pi^2t^2+\left(2^{\frac{b}{\alpha}}\left(3-\frac{b^2B^2\pi^2t^2}{2}\right)-3\right)\log(2)^2\right)\left(\sin((b+1)B\pi t)\right)-\sin((b-1)B\pi t)\right)\right]\right\}\right\}$$

$$c_4(t)=\frac{1}{8}\left\{-\frac{8\sin((\alpha-1)B\pi t)}{\pi t}+\frac{1}{\alpha^2B^2\pi^2t^2+\log(2)^2}\left[4\alpha B\left(\cos((\alpha-1)B\pi t)\log(4)+2\alpha B\pi t\sin((\alpha-1)B\pi t)-\right.\right.\right.$$

$$\left.\left.2^{\frac{b}{\alpha}}\left(\cos((b-1)B\pi t)\log(2)+\alpha B\pi t\sin((b-1)B\pi t)\right)\right)\right]+$$

$$\frac{1}{b^4B^4\pi^5t^5}\left\{32\left[3\left(2^{\frac{b}{\alpha}}-1\right)\left(\sin(B\pi t)+\sin((b-1)B\pi t)\right)+\right.\right.$$

$$bB\pi t\left(\left(2^{\frac{b}{\alpha}}-1\right)\left(-\frac{b^2B^2\pi^2t^2}{2}-3\right)\cos((b-1)B\pi t)+\right.$$

$$\left.\left.\left.\frac{1}{8}b^3B^3\pi^3t^3\sin(B\pi t)+\frac{1}{2}bB\pi t\left(3+2^{\frac{b}{\alpha}}\left(\frac{b^2B^2\pi^2t^2}{4}-3\right)\right)\sin((b-1)B\pi t)\right)\right]\right\}+$$

$$\frac{1}{b^4B^4\pi^5t^5}\left\{32\left[-\left(-1+2^{\frac{b}{\alpha}}\right)bB\pi t\left(\frac{b^2B^2\pi^2t^2}{2}-3\right)\cos((1+b)B\pi t)+\left(3\cdot 2^{\frac{b}{\alpha}}-3-\frac{b^4B^4\pi^4t^4}{8}\right)\sin(B\pi t)+\right.\right.$$

$$\left.\left.\left(3-3(2)^{\frac{b}{\alpha}}+\frac{3}{2}\left(2^{\frac{b}{\alpha}}-1\right)b^2B^2\pi^2t^2-\frac{1}{8}\left(2^{\frac{b}{\alpha}}-2\right)b^4B^4\pi^4t^4\right)\sin((1+b)B\pi t)\right]\right\}+$$

$$4\left\{\frac{2(\sin((1+\alpha)B\pi t)-\sin((1+b)B\pi t))}{\pi t}+\frac{1}{\alpha^2B^2\pi^2t^2+\log(2)^2}\left[\alpha B\left(-\cos((1+\alpha)B\pi t)\log(4)-\right.\right.\right.$$

$$2\alpha B\pi t\sin((1+\alpha)B\pi t)+$$

$$\left.\left.\left.\left.2^{\frac{b}{\alpha}}\left(\cos((1+b)B\pi t)\log(2)+\alpha B\pi t\sin((1+b)B\pi t)\right)\right)\right]\right\}\right\}$$