Scheduling Sequential Processes in Tree Networks

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A b s t r a c t – In this paper we focus on the problem of scheduling time-critical data flow over a measuring tree network. We assume that data are operated in discrete time and their arrival and deadline processes are arbitrary. Our goal is to determine a policy which transmits data with minimum extension time at every node (every link) in a tree network. The models existing in the literature do not consider simultaneous performance of various hardware components of a complex system. When a hardware component of the system fails, the system reconfiguration is often less than perfect. On this assumption we introduce an algorithm to model the availability of measuring systems with colored Petri nets (CPN). Regarding the fact that the availability of a measuring cell i (or branch in the network) is calculated with Markov chains, we model our system with stochastic CPN. The utility of our approach in alleviating the computational burden of measuring systems availability is illustrated via a Markov chain structure.
Key-words: Tree networks, optimal scheduling, colored Petri nets, Markov chains.

1. Introduction

Many systems, and particularly communication networks, are formed by a collection of agents which cooperate using a production schema and compete for resources. The cooperation corresponds to the process plan: each agent in the system transforms some items that are consumed by others in some prescribed fashion to obtain the final products. Competition is introduced by technical or economical restrictions: the different agents share some scarce resources (sensors, switches, communication lines) to perform their tasks. There are several applications of packet switched communication networks where a high variability in packet delivery delay is undesirable. In packetized voice communication systems for example, the quality of signal degrades when the end-to-end delay exceeds a prescribed threshold. In networks carrying control information, a packet incurring a delay larger than the time within which the system state changes becomes useless for control purposes . An important problem in these systems concerns the design of network controls so as to minimize the number of packets reaching the destination after a prescribed threshold. The complex measurement systems include a set of sensors, transducers, multiple part routing due to alternate sequencing in the processing of data. When a device, sensor, transducer or any other hardware component of the system fails, the system reconfiguration is often less than perfect. The notion of imperfection is called imperfect coverage, and it is defined as probability c that the system successfully reconfigures given that component faults occur. The imperfect repair of a component implies that when the repair of the failed component is completed it is not "as good as new". A dependability for evaluation of performance of a manufacturing system is presented. The meaning of dependability here is twofold [3-4]:

- System availability and reliability
- Dependence of the performance of measuring system on the performance of its individual physical subsystems and components.

The model considers the task-based availability of a measuring system, where the system is considered operational as long as its task requirements are satisfied, that means that the system data processing capacity exceeds a given lower bound. In this paper we model the measuring system with stochastic colored Petri nets. In our assumption the availability of a measuring cell i (i = 1,2,…,n, where n is the total number of part type cells in the measuring system) is calculated with a Markov chain which includes the failure rates, repair rates, and coverability of the respective devices in the measuring cell i. The color domains of transitions that load cell i include colors that result in a value between 0 and 1, and the biggest value designates the cell which will transmit data to the root node of the tree network of the measuring system. From the point of view of queuing theory, networks with tree topology are a first step toward an effort to generalize results for single queues. In section 2, the model of the tree network is described, in section 3 is shown the model of a measuring system, and the Markov model of a measuring cell is given in section 4.

2. Transmission scheduling in a tree network

Suppose that data measured by sensors arrive at any of the nodes of a tree network with root node D, with T links between each pair of directly connected nodes, and are designated for node D. There is a deadline and an extinction time (arrival time + deadline) associated with each message, and a message has to reach the destination before its extinction time expires. If the extinction time of a message expires while it is waiting for the repairing of a damaged device in the system, or is being transmitted to an intermediate node, then the message is considered lost and the system reconfigures itself taking into account another message from a downstream node. We wish to find a policy for scheduling the transmission of messages that minimizes the total number of lost messages. We assume that in the tree network the distance (in number of hops) between each node and the root node is known. We also assume that the system is discrete (e.g., is slotted). At slot t the optimal policy would never transmit a message with extinction time strictly less than t+k at a node that is k hops away from D, as this message will be lost. A message at a node k which hops away from D is eligible [4] for transmission at time t if its extinction time is at least t+k. In [5] it is shown that the policy which transmits the eligible messages with the shortest extinction time at every node minimizes the number of lost messages over any time interval. This is referred as the Shortest Time to Extinction (STE) policy. In [6] it is proven the following theorem in order to complete the trivial implementation of the STE policy in a distributed manner, once a node knows its distance from the root node D.
Theorem: For every scheduling policy p

$$T^{STE}(t) \leq T^{p}(t),\ \forall\ t \geq 0. \qquad (1)$$

In the given theorem, it is supposed that the number of links between any two directly connected nodes of the network is identical. The next example shows the necessity of this requirement. Consider the tandem network of Fig.1 [6].

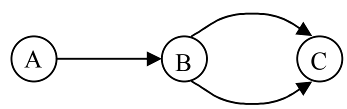

Fig.1 A tandem network

Initially, there are two messages with extinction times two and three units at node A. Messages arrive at the system only in slot [0,1); one to node A with extinction time three units and two to node B, each with extinction time two units. Let p be the policy which transmits the message with extinction time three units at node A at time zero and schedules according to the STE policy at all other times. It is proven in [6] that p loses one few message than STE.

3. Modeling the measuring schedule

We will assume that the reader is familiar with colored Petri net theory and their application to manufacturing systems or we refer the reader to [2]. Each part entering the system is represented by a token. The color of the token associated with a part has two components [4]. The first component is the part identification number and the second component represents the set of possible next operations determined by the process plan of the part. It is the second component that is recognized by the colored Petri net model of the cell, and the first component is used for part tracking and reference purposes. Let B_i be a (1xm) binary vector representing all the operations needed for the complete processing of part type i. Let E_i be a (mxm) matrix representing the precedence relations among the operations of part type i, where m is the number of operations that are performed in the cell. For a part to be processed in the cell it requires at least one operation that can be performed in the cell, that implies $B_i > 0$. Also, for a part type where there is no precedence relationship between operations required, E_i is a matrix of zeros. For a part with identification x and part type y, the initial color of the corresponding token is:

$$V_{yx} = [yx, B_y - (B_y \,.\, E_y)] \qquad (2)$$

Where ($B_y \cdot E_y$) is a matrix multiplication.
For example consider the process plan of part type L_1 shown in Figure 1.

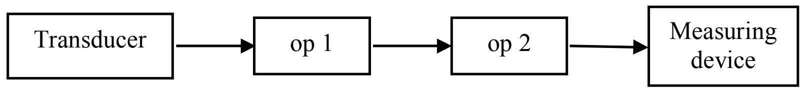

Fig.2 Process plan for part type L_1

Our process plan first requires operation op 1 and then op 2 for complete processing. We assume that our FMS can complete 5 different types of operations. For part type L_1, we have: B_{L1} = [0 0 0 1 1]

$$E_{L_1} = \begin{array}{c|ccccc} & op5 & op4 & op3 & op2 & op1 \\ op5 & 0 & 0 & 0 & 0 & 0 \\ op4 & 0 & 0 & 0 & 0 & 0 \\ op3 & 0 & 0 & 0 & 0 & 0 \\ op2 & 0 & 0 & 0 & 0 & A_2 \\ op1 & 0 & 0 & 0 & A_1 & 0 \end{array}$$

Where A_1 and A_2 are the availability of measuring cell 1 (which performs operation 1), respectively the availability of measuring cell 2 at time t. The availability A_i of cell i is calculated, as shown in Section 4, with Markov chains. We notice that A_i is reevaluated at each major change in the process plan of FMS (such as occurrence of events: damages of hardware equipments, changes of process plan, etc). Assuming that $A_1 > A_2$, then we assign to A_1 value 1 and to A_2 value 0, so that applying (2), the initial color of the token corresponding to a part that belongs to part type L_1 with identification 1 would be $V_{L1.1}$ = ($L_{1.1}$, 00001). Note that the information carried in the initial color indicates the first (next) operation(s) to be performed. Generally, we may say that V is the set of colors that represent all the possible combinations of operations that can be performed in the mesuring system. Each member of set V is a vector of n components, where n is maximum number of operations performed in the cell. For example, in a FMS with 5 operations to be performed, we may have V = {00000, 00001, …, 11111}. For simplicity, we assumed that the function which maps operations to measuring devices on which these operations can be performed is modeled in the associated CPN with places, labeled with the operation identification number.

4. Modeling the measuring cell system

The requirement for measuring cell i, including N_1 devices of type M_i, is that at least k_i of these devices must function for the system to be operational. To determine the system availability which includes imperfect coverage and repair, a failure state due to imperfect coverage and repair was introduced [3]. To explain the impact of imperfect coverage in measuring, consider the system that includes two identical measuring devices (Fig.3). If the coverage of the system is perfect, i.e., c = 1, then operation op 1 is performed as long as one of the devices is operational.

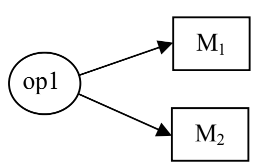

Fig.3 The measure system with two identical devices

If the coverage is imperfect, then operation op 1 fails with probability 1 – c, if one of the devices in Fig.3 fails. We may say that, if operation op 1 has been scheduled on device M_1 that has failed, then the system in Fig.3 fails with probability 1 – c. The Markov chain for machine cell i is shown in Fig.4. The coverage of the cell in Fig.4 is c and successful repair factor is r. At state k_i, cell i is functioning with only k_i devices operational.

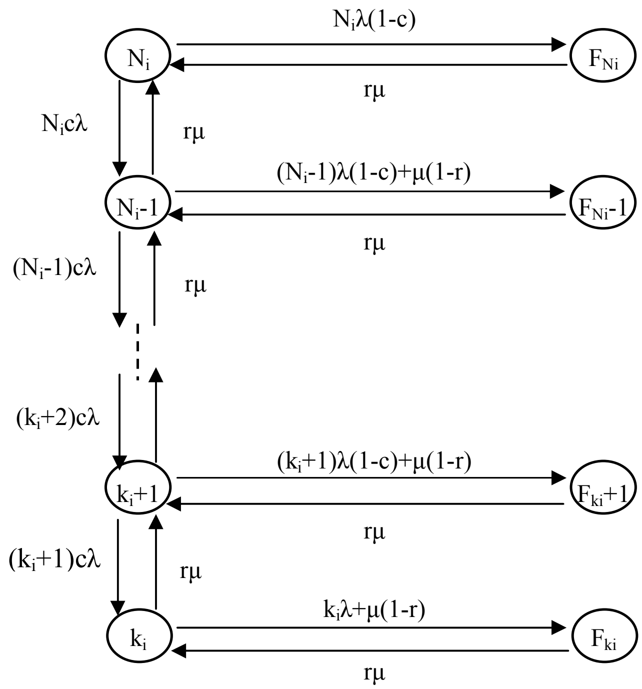

Fig.4 Markov model for cell i

At state N_i cell i is functioning with all N_i devices operational. The state of cell i changes from working state w_i, for $k_i \le w_i \le N_i$, where w_i is the number of operational devices in cell i, to failed state F_i , either due to imperfect coverage (1-c) or due to imperfect repair (1-r). If the fault coverage of the system and repair of the components are perfect, the Markov chain in Fig.4 reduces to a one-dimensional model [7]. The solution of the Markov chain model in Fig.4 is a probability that at least k_i devices in cell i are working at time t. Availability formula for cell i is given in the next relation:

$$A_i(t) = \sum_{w_i = k_i}^{N_i} P_{k_i}(t), \text{ for } i = 1, 2, \ldots, n \qquad (3)$$

Where: $A_i(t)$ = availability of cell i at time t;

$P_{k_i}(t)$ = probability of k_i devices being operational in cell i at time t;

N_i = total number of devices of type M_i in cell i;

k_i = required minimum number of operational devices in cell i.

After a Markov chain for each cell of the measuring system is constructed and desired probabilities $A_i(t)$, i = 1, 2, …, n corresponding to machine cell are determined, the stochastic colored Petri net can be initialized, and the simulation process of FMS begins. In Fig.4 the parameters λ, μ, c, r denote respectively the failure rate, repair rate, coverage factor, and the successful failure repair rate of a measuring equipment. The first part of the horizontal transition rate with the term 1-c represents the failure due to imperfect coverage of alternative equipment. The second part, with the term 1-r represents imprecise repair of the hardware components. The vertical transitions reflect the failure and repair of the equipments. We assume that only one device fails at a time, in a certain operation cell.

5. Conclusions

In this paper we have proposed a new architecture to model a large class of measuring systems using stochastic colored Petri nets. Advantages of this approach are:

- Alternate sequencing of operations is allowed during processing;

- Device assignments for operations are made dynamically during processing;

- The model of the measuring system created captures all possible operation sequences in the system.

An analytical technique for the availability evaluation of measuring systems was also presented in this paper. The advantages of this approach are:

- The construction of large Markov chains is not required, and also;

- It incorporates imperfect coverage and imperfect repair factors in the Markov models;

- It reveals when the system coverage and the component repair are not perfect;

- It allows determination of the timing of a major repair policy of the systems.

Further researches will focus on modeling measuring systems with semi-Markov processes.

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