Modeling the Railway Data Transmitter CN-75-6 with Markov Chains

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Abstract

This paper focuses on analysis of randomized pulse width modulation schemes based on finite Markov chains. Randomized modulation of switching in power converters reduces filtering equipments and allows an explicit control in time-domain performance. Randomized modulation is very effective for narrow-band constraints; therefore it is proper to be implemented into the command of the railway data transmitter for the traffic security.
Keywords: *Markov chains, pulse width modulation, data transmitter.*

1. Introduction

Typically, a data transmitter consists of command logic and a switching power converter. The switching power converter uses only switching devices, energy storage elements, and relays an appropriate modulation of the switches to convert the available a.c. or d.c. voltage (current) waveforms of the power source into the a.c. or d.c wave forms required. The switches are generally semiconductor devices: diodes, thyristors, IGBT, etc. [1]. A switching scheme involves generating a switching function f(t), which by definition has the volume one when the switch is conducting, and the zero value otherwise. This is schematically indicated in Fig.1.
Since power converters generally operate in a periodic steady state, transmitter wave forms of interest (in the electronic code transmitter CN-75-6) are typically periodic functions of time, in the steady state, as illustrated in Fig.2. The structure of the codified pulses of the CN-75-6 transmitter involves 10 sine waves of 75 Hz frequency followed by a pause virtually carrying a multiple of 10 sine waves of 75 Hz frequency. Because the information is carried by pauses, the CN-75-6

decodificator counts the number of sine waves periods which fits the length of the pause.

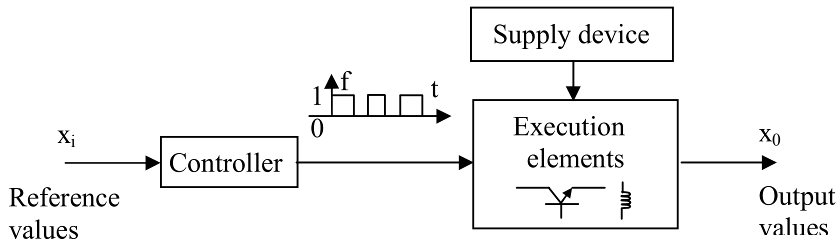

Fig.1 Schematic representation of a switching transmitter

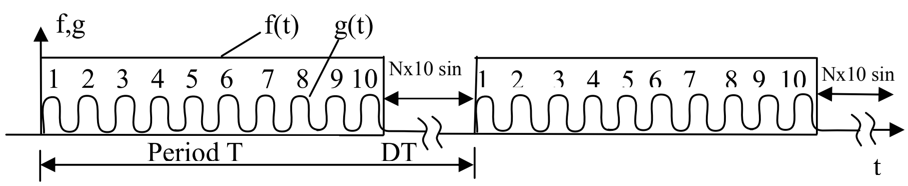

Fig.2. Switching functions of the electronic code transmitter CN-75-6

The resulted number, which must be a multiple of the sinusoids, indicates a specific command for the railway installations, such as a color displayed by the traffic lights. As shown in Fig.2 the switching function f(t) determines the structure of the codified pulses g(t) of the CN-75-6 transmitter [2]. The function f(t) given by the controller (Fig.1) reflects steady-state wave forms for the controlled voltages, respectively for the controlled code.

It is well known that the average value, or duty ratio D, of f(t) usually determines the nominal output of a d.c./d.c. converter, while the fundamental component of f(t) usually determines the output of a d.c./a.c. converter; similar statements can be made for a.c./d.c. or a.c./a.c. converters.

Converters wave forms that are periodic have spectral components only at integer multiples of the fundamental frequency. The allowable harmonic content of some of these waveforms is constrained: our circuit, type CN-75-6, ideally should have only the 75 Hz fundamental component present. In this case, stringent filtering requirements may be imposed to the railway insulation resistance, and on the power converter. As the use of pulse width modulation (PWM) in power converters controlled by microprocessors evolved, new methods became available to address the effects of electromagnetic interference (EMI). While an effort was directed toward the optimization of deterministic PWM waveforms, an alternative in the form of randomized modulation for d.c./a.c. and a.c./a.c. conversion is based on schemes in which successive randomizations of the periodic segments of the switching pulse train are statistically independent and governed by probabilistic rules. These schemes are denoted as stationary [3]. In this paper we describe an approach to the synthesis of this class of stationary randomized modulation schemes that enables explicit control of the time-domain performance of the CN-75-6 railway data transmitter, used in the Romanian railways system.

2. Switching synthesis with Markov chains

The basic synthesis in randomized modulation is to design a randomized switching procedure that minimises given criteria for spectral characteristic of f(t), while respecting time-domain behavior constraints. Practically optimization procedure assumes the minimization of discrete spectral components (denoted as narrow-band optimization, [4]), and the minimization of signal power in a given frequency range (denoted as wide-band optimization, [5]). The case of pulse trains specified by periodic Markov chains is denoted as ergodic cyclic [4], [5]. We assume that the state of the chain goes through a sequence of n classes of states C_i , occupying a state in each class for an average time σ_i, i = 1, ..., n.
The time-average autocorrelation [6] of a random process f(t) is defined as:

$$R_f(\tau) = \lim_{n\to\infty} \frac{1}{2w} \int_{-w}^{w} E[\, f(t) \cdot f(\tau + t)] \cdot dt \qquad (1)$$

where the expectation E[.] refers to the whole ensemble [.].

The contribution that states the Markov chain belonging to the class C_k, with the time-averaged autocorrelation (1) is scaled by $\tau_k / \sum_{i=1}^{n} \tau_i$, where τ_i is the expected time spent in the class C_i before a transition into the class C_{i+1}. We define P as the n x n state-transition matrix, and its (k,i)th entry is the probability that at the next transition the chain goes to state i, given that it is currently in state k. Each row of P sums to 1; P is thus a stochastic matrix, and therefore has a single eigenvalue λ_i = 1, with corresponding eigenvector 1_n = [1 1 ... 1], and all other eigenvalues with moduli strictly less than one. It can be shown [1] that after a possible remembering of the states, the matrix P for a periodic Markov chain can be written in a block-cyclic form.

$$P = \begin{bmatrix} 0 & P_{12} & 0 & \dots & 0 \\ 0 & 0 & P_{23} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & P_{n-1,n} \\ P_{n1} & 0 & 0 & \dots & 0 \end{bmatrix}$$

Let P_P denote the product of submatrices of P: $P_P = P_{n1} \ldots P_{23} . P_{12}$, and let v_i denote the vector of steady-state probabilities, conditional on the system being in class C_i. Then, we have:

$$V_i{*} = v_i \cdot P_P \tag{2}$$

The average time spent in class C_i is: $\tau_i = \sum_k V_k^* \cdot \tau_k$ (3)

We notice that in relation (3) the summation refers to all states in class C_i.

Let $T_o = \sum_{i'1}^{n} \tau_i$ and let $\Theta_i = \text{diag}\ (V_i{*})$. If the first pulse belongs to the class i, then the pulse $\tau_i + \tau$ belongs to the class (i + m) / mod n, where m represents the number of transitions between pulses τ_I and $\tau_i + \tau$.

We mention that the average duration of some classes may be null, which means that these classes (with corresponding states) are skipped.

This approach allows us to build a simplified Markov chain model (we present this assumption, which we believe to be novel, in the next section). When we add the contribution of all classes to the average power spectrum (scaled by the relative average duration of each class), the result can be written as follows [4].

$$S(f) = \frac{1}{T_0}\left[\sum_{i=1}^{n}\frac{\tau_i}{T_0}\cdot U_i^T(f)\cdot\Theta_i U_i(f) + 2R_e\left(1_n^T\cdot S_C\cdot 1_n\right)\right] + \frac{1}{T_0^2}R_e\left(1_n^T\cdot S_d\cdot 1_n\right)\sum_{i=-\infty}^{\infty}\delta\left(f-\frac{i}{T*}\right) \tag{4}$$

Where T* is the greatest common divisor of all waveform duration, 1_n is an n x 1 vector of 1 and U_i is the vector of Fourier transforms of waveforms assigned to states in class C_i.

The matrix S_C has a Toeplitz structure, with (k,i)th entry

$$S_{ck,i}(f) = \frac{\tau_i}{T_0}\cdot U_k{}^T(f)\cdot\left(I-\Lambda_k(f)\right)^{-1}\cdot\Lambda_{k,i}(f)\cdot U_i(f) \tag{5}$$

There Λ_k is a product of n matrices

$$\Lambda_k = Q_{k-1,k},\ldots,Q_{k,k+1}$$

and $\Lambda_{k,i} = Q_{k,k+1},...,Q_{i-1,i}$

Where Q is a matrix n x n whose (k,i) entry is $Q_{k,i}(\sigma) = P_{k,i}\delta(\sigma-\tau_k)$.
Also, the (k,i)th entry of S_d is given by relation:

$$S_{dk,i}(f) = \frac{\tau_i}{T_0} \cdot U_k^T(f) \cdot V_k \cdot (V_i)^T \cdot U_i(f) \qquad (6)$$

Analogous with the notion of truncated Markov chains with absorbing states [6], we propose a simplified model of Markov chains for random modulation. The proposed Markov chain $X^{(m)}$ has the states {m,m+1,…} aggregated into an absorbing class of states, which has transition matrix ${}_{(m)}T$ satisfying:

$$_{(m)}T = \begin{bmatrix} _{(m)}P & p_m \\ 0 & 1 \end{bmatrix} \qquad (7)$$

Where $p_m = \left[\sum_{j \geq m} p_1 j, \sum_{j \geq m} p_2 j, ..., \sum_{j \geq m} p_{m-1,j} \right]$. Since ${}_{(m)}P$ is irreducible for all m, it follows that $X^{(m)}$ constitutes an irreducible Markov chain for all m [6]. The states {1,…,m-1} form a transient set and m is an absorbing class of states. The n-step transition probability matrix ${}_{(m)}T^n$ can be written as:

$$_{(m)}T^n = \begin{bmatrix} _{(m)}P^n & \left(I_m + {}_{(m)}P + {}_{(m)}P^2 + ... + {}_{(m)}P^{m-1}\right)p_m \\ 0 & 1 \end{bmatrix} \qquad (8)$$

Where ${}_{(m)}P^n = {}_{(m)}p_{ij}^{(n)} = p_{ij}(n)$.
The transition probability between realizable classes of states is 1. This probability also indicates the priorities between the states of the system.

3. Example of pulse width modulation governed by a periodic Markov chain

In this example, applicable to railway data transmitter CN-75-6, our goal is to generate a switching function in which blocks of pulses have deterministic duty ratios: [0.5, 0.75, 0.5, 0.25]. The periodic Markov chain shown in Fig.3, with eight states divided to four classes, is an example of a solution to such a problem [6], [7]. A short (duration 3 /4) and a long (duration 5/4) cycle is available in each of the four classes.

According to the theoretical approach in the previous paragraph, we build a simplified Markov chain in Fig.4. The Markov chains in Fig.3, respectively in Fig.4 have the same transition probabilities between states S_i, i=1,…,8 of classes C_j, j=1,…,4. We notice that the Markov chain in Fig.4 is more intuitive and tidy than the one in Fig.3. The transition probabilities equal to 1 in Fig.4 are conditioned by the existence of transition probabilities between the states of different classes.

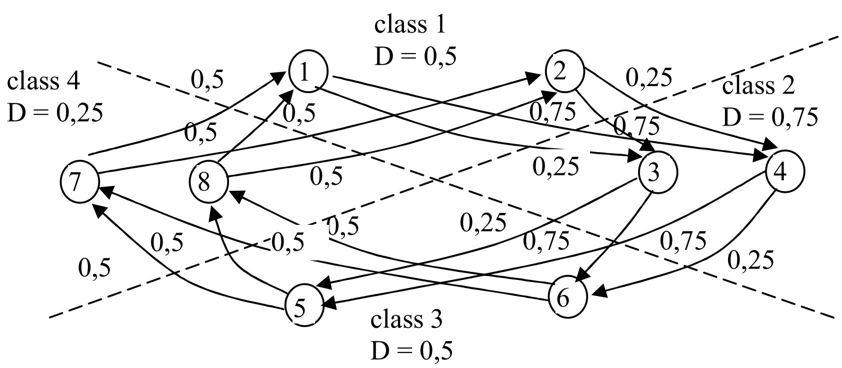

Fig.3 Classic Markov chain for modeling the switching example

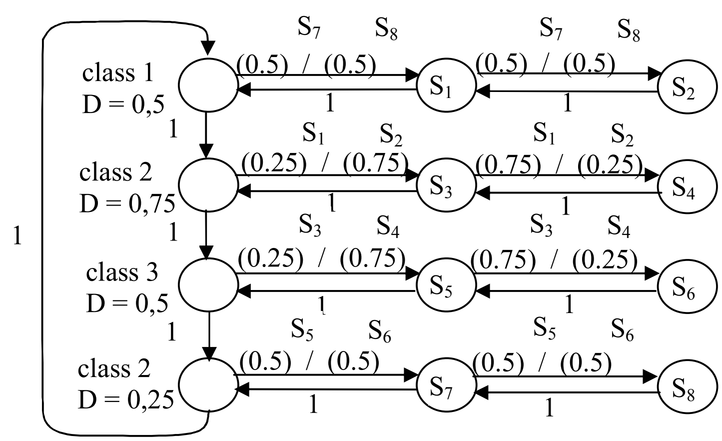

Fig.4. Proposed Markov chain for modeling the switching example

The above given duty ratios in the present example and modeled with Markov chains in Fig.3, respectively in Fig.4, represent the switching between colors red, yellow, red, green displayed by railway traffic lights commanded by data transmitter CN-75-6. We analyze this Markov chain with equation (4) and we compare the theoretical predictions with the estimates obtained in Monte Carlo simulations. The agreement between the two is quite satisfactory: the theoretical prediction for the impulse strength at f = 4 is 0.0036, and the estimated value is 0.0037. The Markov chain represented in Fig.4 allows dealing with many more classes of states because graphical representation is simplified and can significantly improve the tractability of the optimization of the Markov chains with many states.

4. Conclusions

Our paper focuses syntheses results for randomized modulation strategies achieved with Markov chains, suitable for power converter used in the railway data transmitter CN-75-6. Randomized modulation switching schemes governed by Markov chains applicable to d.c./a.c. or d.c./d.c. converters have been described. Our representation for complex periodic Markov chains we believed to be novel. We also believe that this new model offers a new perspective for the spectral characteristics and other associated waveforms in a converter to the probabilistic structure that governs the dithering of an underlying deterministic nominal switching pattern. Further research will continue to focus on minimization of one or multiple discrete harmonics. This approach corresponds to cases where the narrow-band characteristics corresponding to discrete harmonics are harmful, as for example in the railway traffic security.

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