Availability of Fluid Stochastic Event Graphs

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Abstract— **Petri nets provide a compact and graphical way to model large and complex discrete event systems (DES). For such systems, the state-space explosion is problematic. Fluid stochastic event graphs are decision free Petri nets, which can represent systems with failures. This paper presents an estimation algorithm for state space estimation and optimization of failure-prone DES.**
Keywords– **Discrete event systems, fluid Petri nets, space estimation.**

I. INTRODUCTION

According to the complexity of modern technological processes, many model analysis and control algorithms are based on the model state-space sizes. In this paper we propose, based on the works in [1] and [2], an algorithm for analyzing and optimizing manufacturing systems subject to failures. It is based on the fluid approximation of a class of Petri nets and called fluid-event graphs. In a fluid/event graph, places hold fluids instead of discrete tokens. Transitions fire continuously, drawing fluids out of its input places and injecting fluids into its output places. Firing speed of transition is limited by a maximal speed. For failures systems, transitions can be either in operating state or in failure-state. The discussed systems are hybrid, that means they have discrete-event components characterized by failures and repair of transitions, and continuous components characterized by markings and transition firing speeds. We assume that the discrete-event component does not depend on the continuous component [2]. This assumption is related to the time dependent failures of the failure – Prone manufacturing systems. The following users for the state–space size estimation can be mentioned:

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- Evaluating the trade-off between model detail and solution complexity. This is necessary because, at some point,

dependent on the specific model, additional computation time is not worth the improvement in accuracy.
- Determining the appropriateness of a particular analysis technique. Algorithms optimized for “small” or “large” problems can be applied appropriately.

The problem of state–space size estimation of Petry nets (PN) is being pursued in two manners: top-down and bottom–up [3], [4]. At the expense of complete generality, the bottom–up approach offers better accuracy.

II. FLUID PETRI NETS

Fluid Petri nets are an extension of classical Petri nets [5]. In fluid PN (FlPN), marking a place is a real number called the token content. Also, because transitions fire continuously according to some firing speed, we shall say firing speed instead of firing sequence. In fluid Petri nets, we associate to each transition a maximal firing speed, and flowing notations are used:

A_i = maximal firing speed of transition i;
a_{it} = firing speed of transition I at time t;
m_{it} = marking of place p_i at time t;
m_{oi} = initial marking of place p_i ($m_{io} = m_{oi}$);
q_{it} = cumulative firing quantity of transition I up to time t.

A control policy a_{it} it is feasible if $m_{it} \geq 0$

A transition can fire at its maximal firing speed if each of its input places has positive markings. A generally firing policy for fluid Petri nets it is [2]:

$$a_{it} = \min_{Tj/m_{it}=0} A_j \quad , \ \forall i \in \{T\} \qquad (1)$$

Where T_j is the transition which follow the place p_i with marking m_{it}. {T} is the set of transitions in the considered fluid Petri net.

For example let us consider the fluid Petri net in Fig.1.

In Fig.1 consider the place p_1 and assume that $A_1 \geq A_2$. Then transition T_2 can fire at its maximal speed and the fluid level of place p_1 evolves as shown in Fig.2.a.

If $A_1 \geq A_2$, then T_3 fires at its maximal speed until place p_1

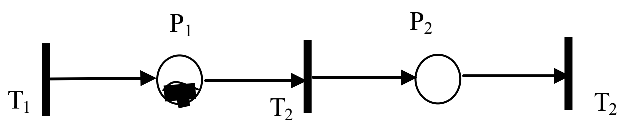

Fig.1. An example of fluid Petri net

becomes empty, and then it fires at reduced speed A_1.
Fig.2.b. illustrates the evolution of the fluid level. In general, the fluid level of each place evolves piecewise linearly according to the firing speed of its input / output transitions.

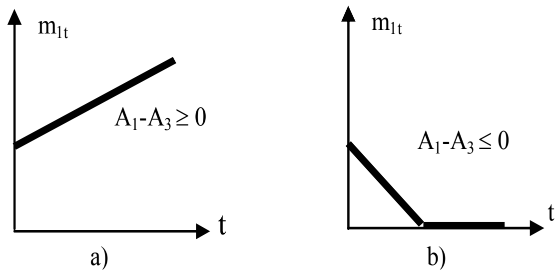

Fig.2. Evolution of fluid Petri net given in Fig. 1

In the following analysis we consider a state estimation function (Se-function) that describes the net's state-space size, that has A_i maximal firing speed of transition i, i=1,…,n, n being the number of places in the net. A Se-function has the following properties:

1) It is a function of A_i only. Thus, the influence of any control tokens must be invariant with respect to A_i, $i=1,\dots,n. \in \{T\}$

2)se(Aj)=0 implies the subnet cannot possibly contain Aj maximal firing speed of transition $j \in (1,\dots,n) \in \{T\}$.

III. SE – FUNCTIONS FOR BASIC CONFIGURATIONS OF FLPN

In this paper, we determine the size of the state-space for the underlying system, irrespective of his dimension, by decomposing it into several basic configurations e.g., subnets (SN) that can form complex (large) FlPN, when combined interconnections for mechanisms of execution and failure/repair are presented here. FlPN can model the execution of sequential, parallel and choice operations. Fig. 3 illustrates two subnets in series, whereby fluid passes from SN_1 to SN_2.

The r tokens possible distribution among the places corresponding to the subnets SN_1, and SN_2 is due to the fluid repartition among the respective places, separated by transitions T_i, I=1,2,3, with a_{it} firing speed.

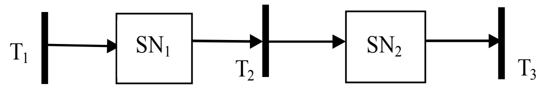

Fig.3. Series basic interconnection of FlPN

The interconnection Se–function is determined from the fact that tokens can be distributed between the two subnets according to the following relation [2], [6]:

$$Se_{series} = \sum_{i=0}^{r} Se_{a_{1t}}(i) Se_{a_{2t}}(r-i) \qquad (2)$$

Where $Se_{a_{nt}}(i)$, the k[th] partition of r, is an n-vector of non-negative integers that sum to r.
A_{it} is the firing speed of transition i, i=1,2, at time t.
For n subnets, the Se-function is:

$$Se_{series} = \sum_{i=0}^{r} Se_{a_{1t}}(i) Se_{a_{2t}}(i-1)..Se_{a_{i-1t}}(r-i+1) Se_{a_{it}}(r-i) \qquad (3)$$

Parallel execution of operation is depicted in Fig.4. For every token entering through T_1, there is one in each subnet. The Se-function is given by relation (4):

$$Se_{parallel} = \prod_{i=1}^{n} Se_{a_{it}}(r) \qquad (4)$$

The choice among sub-nets is depicted in Fig.5. There are three constructs to consider: SN_1, SN2, and, as a group the places P_1 and P_2. Having r tokens among P_1 and P_2 generates $C_r^2 = r+1$ states.
For n sub-nets, we have:

$$Se_{choice} = \sum_{i=0}^{n} se_{a1t}(i) Se_{a2t}(i-1)..Se_{a_{i-1t}}(r-i+1) Se_{a_{it}}(r-i).(i+1) \qquad (5)$$

FlPN of systems containing unreliable components, may include models for the failure and repair of these components. Fig.6 depicts a common model for these operations. Under normal functioning, T_2 fires instead of T_{fail}, leaving SN_2 out of consideration.

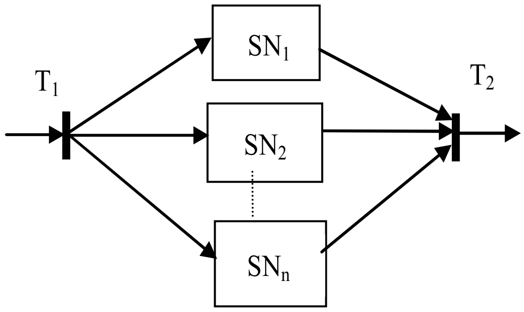

Fig.4. Parallel basic interconnection of FPN

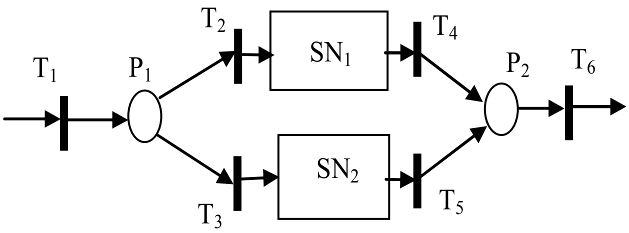

Fig.5. Free choice basic interconnection of FlPN

The choice between T_2 and T_{fail} allows SN_2 to be used by all r tokens. Thus, SN_1 and SN_2 are, in series, producing:

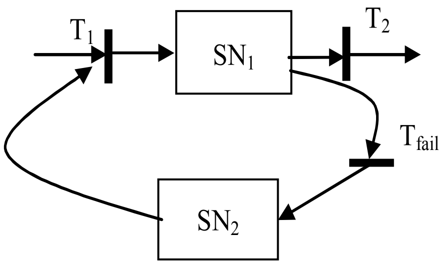

Fig.6. Failure/repair interconnection of FlPN

$$Se_{failrep} = \sum_{i=0}^{r} Se_{a_1t}(i).Se_{a_2t}(r-i) \qquad (6)$$

IV. AN EXAMPLE OF FLPN WITH FAILURE AND REPAIR

In Fig.7 a number of modeling assumptions were made for the convenience of the presentation:

a) The tasks performed by M_2 and R_2 were aggregated into a single transition representing loading and unloading at M_2.

b) The time associated with the unloading tasks performed by R_1 is incorporated into transition T_2.

c) The failure-repair loop is added to M_1.

d) The time delays are associated with transitions only.

e) A Firing speed of a transition equals to the reciprocal of the average firing delay time of the corresponding event or operation.

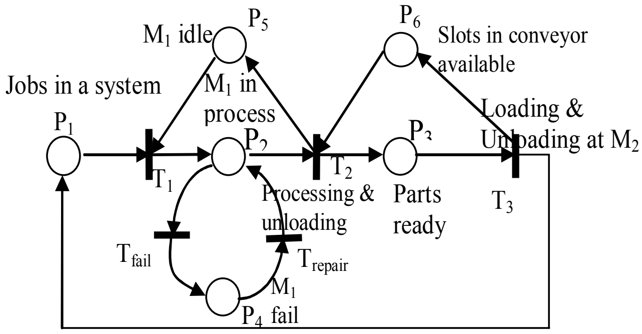

Fig.7. FlPN model for the production line

Since we are interested in the steady-state behavior of the FPN, the past exit and entry path are connected, and the number of parts in the system is limited by the content of place P_6. The FlPN presented in Fig.7 consists of three subnets: $SN_1(P_2T_{fail},P_4,T_{repair})$, $SN_2(T_1,P_2,T_2, P_5)$ and $SN_3(T_2,P_3,T_3,P_6)$. For each of these subnets, we apply the corresponding formula (one of those given above) in order to calculate the Se-function. The global se-function of the FPN in Fig.7 is given by the following relation, according to SN_i, i=1,2,3, connections:

$$Se_{global} = \sum_{i=0}^{r} Se_{SN_1}(i).Se_{SN_2}(i).Se_{SN_3}(r-i) \qquad (7)$$

Where, r is the content of place P_6.

Using (7) and the Se- functions given in (6) for SN_3, respectively in (2) for SN_1 and SN_2, we obtain, as a function of r, the state estimating function for the net given in Fig.7. For example, we suppose (in order to simplify the calculus) that A_j = const = 1, where j=1,…,5 is the number of transitions in Fig.7 (see relation (1)). Thus, we obtain the following Se_{global} for the net given in Fig.7:

TABLE 1. SE FOR THE NET GIVEN IN FIG.7

r	Se_{global}
0	1
1	12
2	84
3	132
. .	. .

CONCLUSION

A bottom-up state-space size estimation technique for fluid Petri nets (FlPN) has been described. The estimation relies on the computation of state estimating functions (Se functions). Several researchers have documented the state-space explosion behavior but there is no direct correlation between the specific behavior of FPN and the used mechanisms such us top-down and bottom-up. A lot of model analysis and control algorithms are based on the model state space, and there it is why they are affected by large state space sizes. Benefits of this approach include simple representation of Se-functions that facilitate automation, and the possibility to interject hand-computed results into the estimation. Errors in the estimation may result from changes in the FlPN model to permit analysis in the Se functions. Further research will include the colored Petri nets into the FlPN, and the calculus of theirs Se-functions.

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